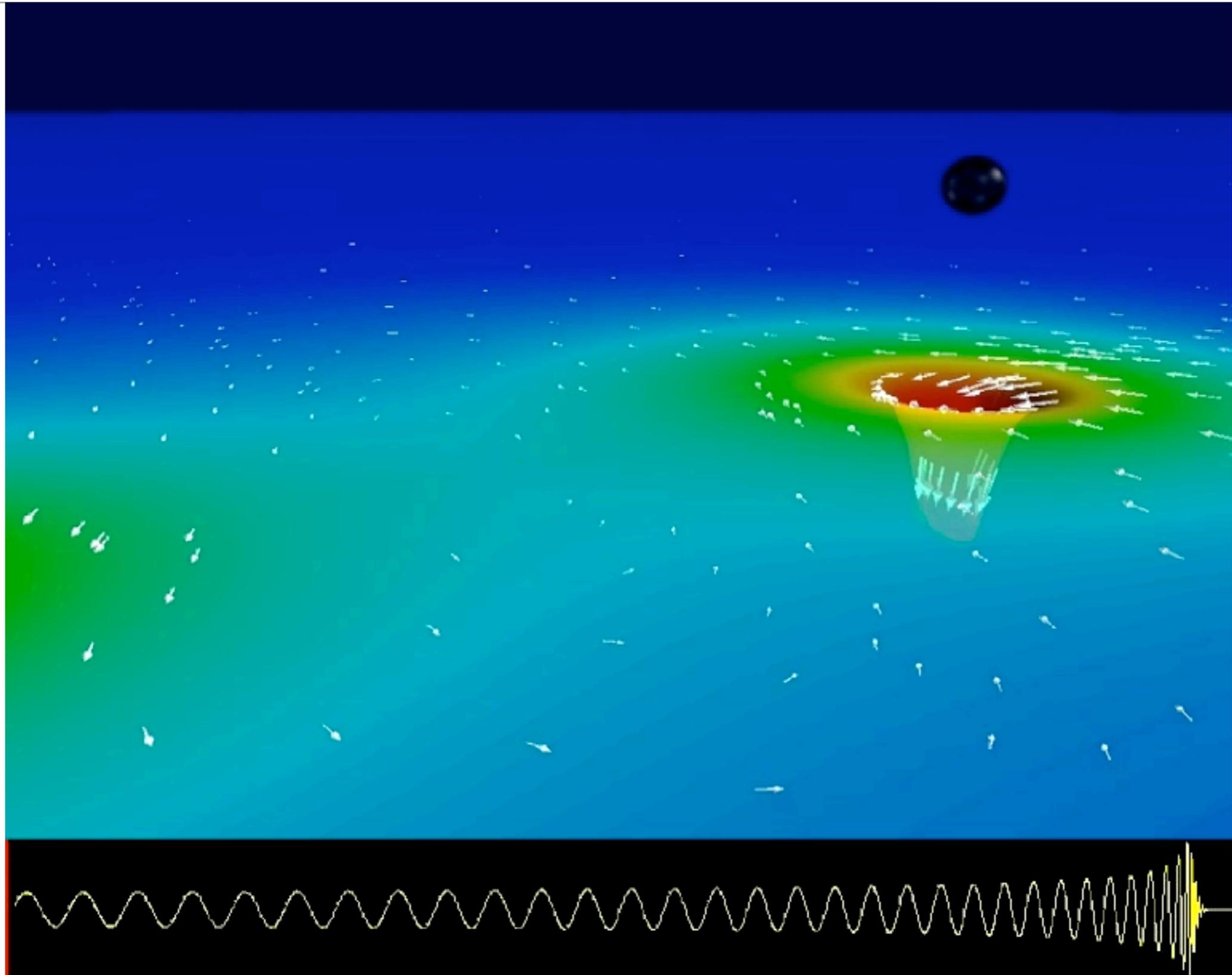


Quantum Mechanics of Macroscopic Objects

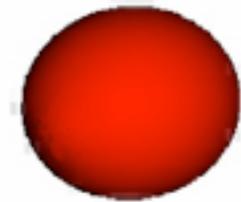
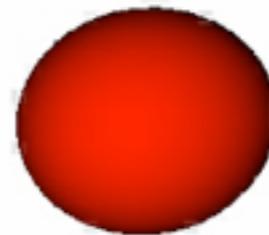
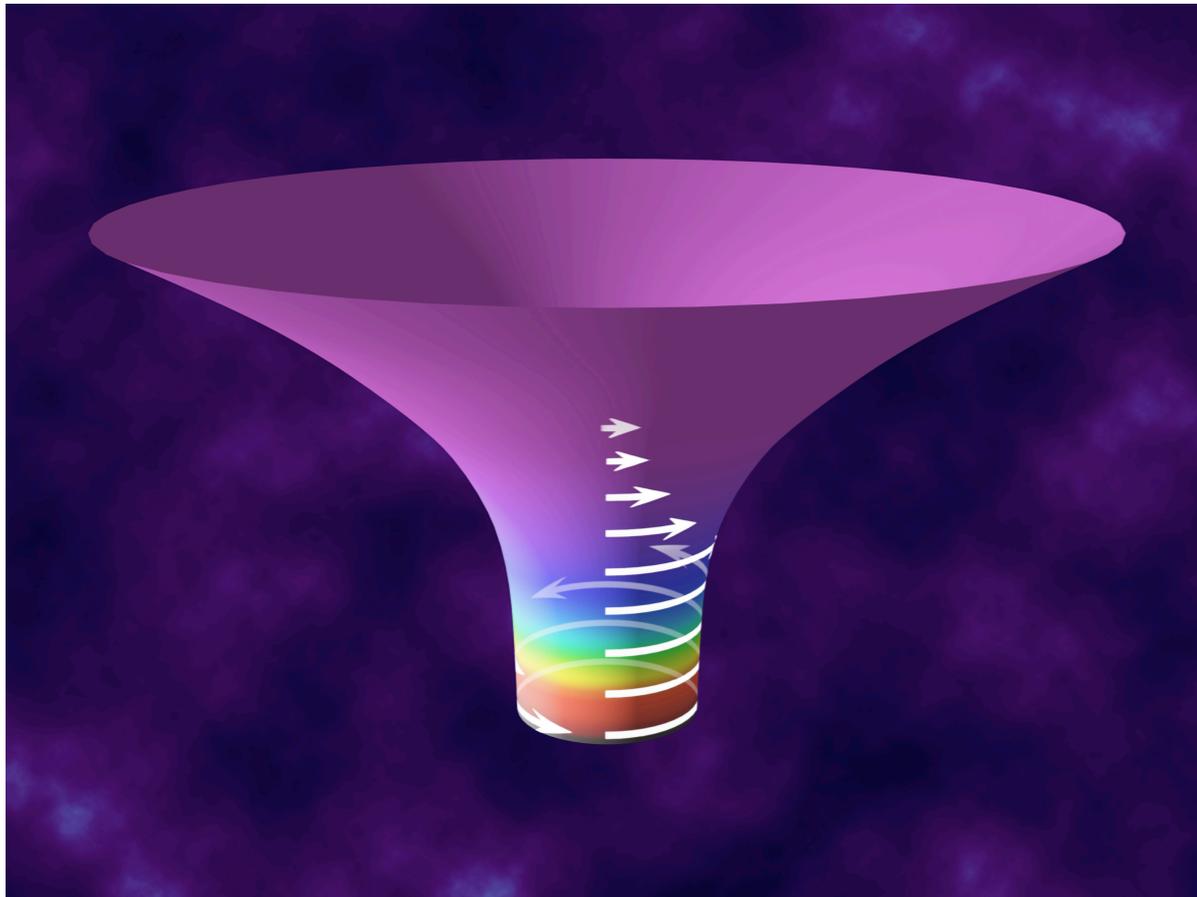
Yanbei Chen

California Institute of Technology

Black Holes Collide

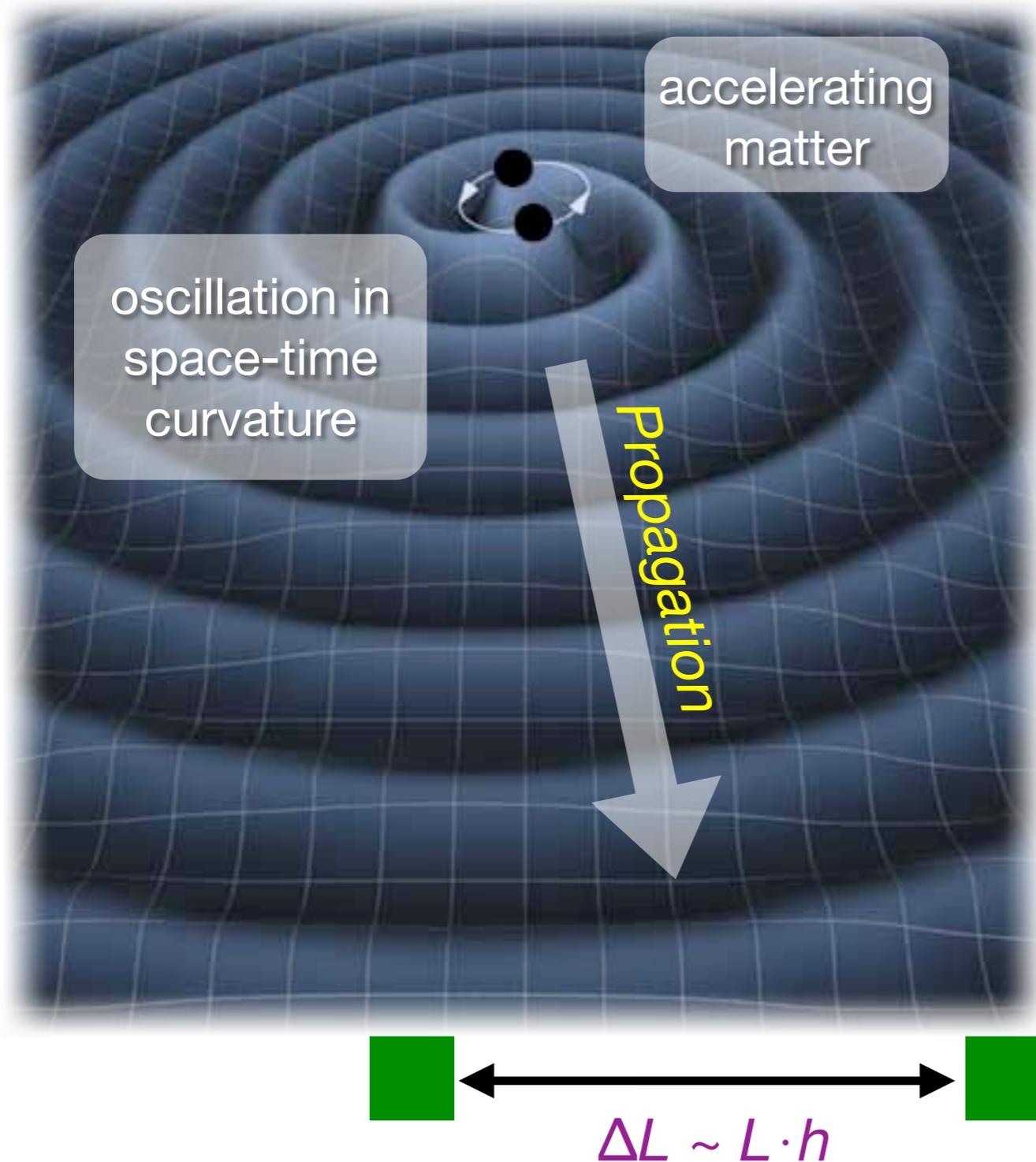


Black Holes



- Black Holes form after very heavy stars run out of fuel, they collapse from **millions of km to several km**
- Time stops at the surface of black holes. Space is highly curved.
- They get distorted when they collide with each other.
- The best way to learn about black holes is to detect **gravitational waves**.

Gravitational Waves are Ripples of Spacetime



- Relative change in distance is

$$\frac{\Delta L}{L} = (\sim 0.1) \frac{\text{size of system}}{\text{distance to earth}}$$

- Black-hole collision events not frequent enough in our own galaxy. Andromeda is **3 Million Light Years** away

$$\frac{\Delta L}{L} = (\sim 0.1) \frac{10 \text{ km}}{3 \text{ Mly}} \sim 4 \times 10^{-20}$$

- If we separate objects by **4 km**

$$\Delta L = 10^{-16} \text{ meter}$$

(still an over estimate)

relative distance between free objects oscillates

Laser Interferometer Gravitational-wave Observatory (LIGO)

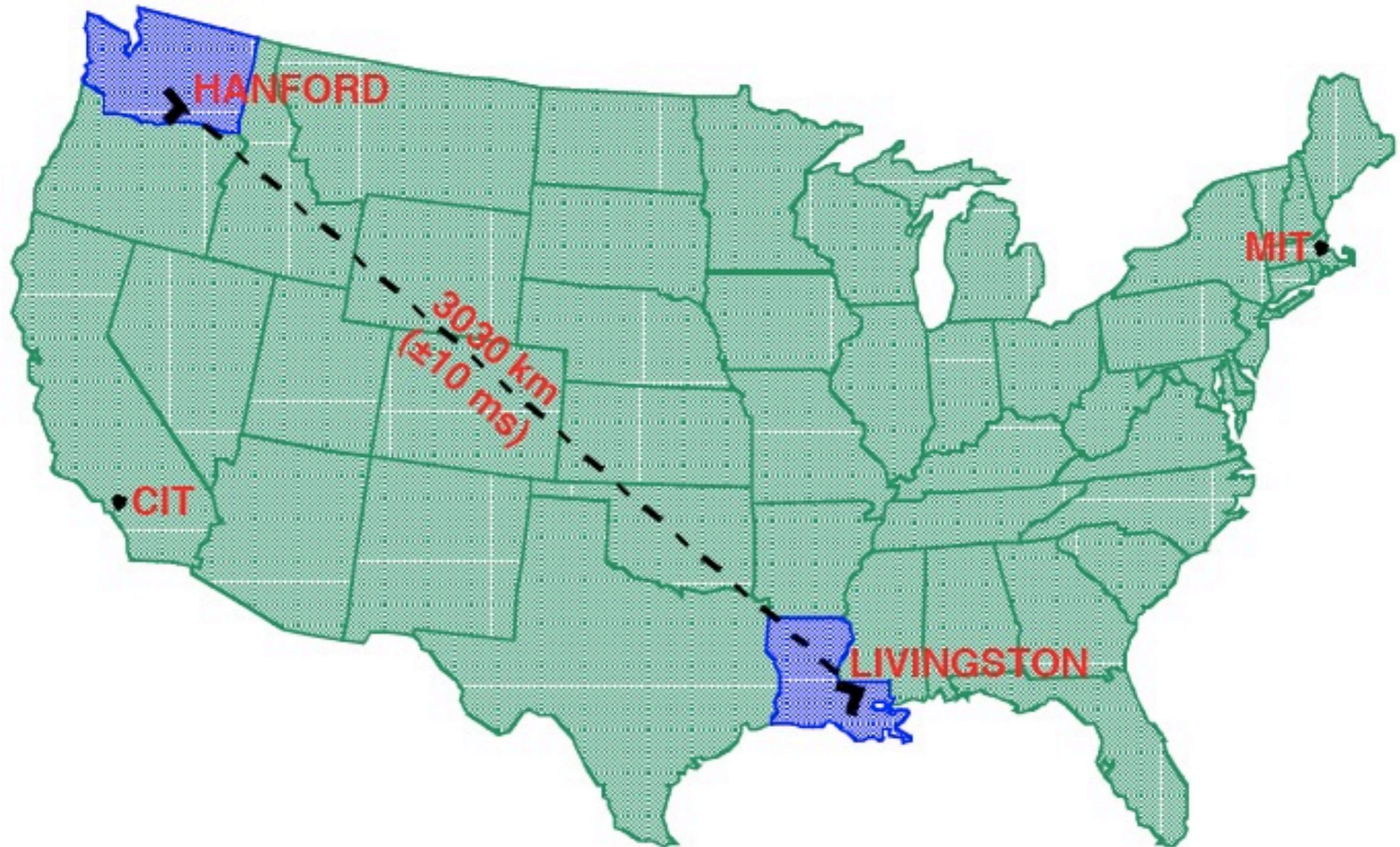


Hanford, Washington

Laser Interferometer Gravitational-wave Observatory (LIGO)



Livingston, Louisiana



International Partners



VIRGO: near Pisa, Italy
French-Italian

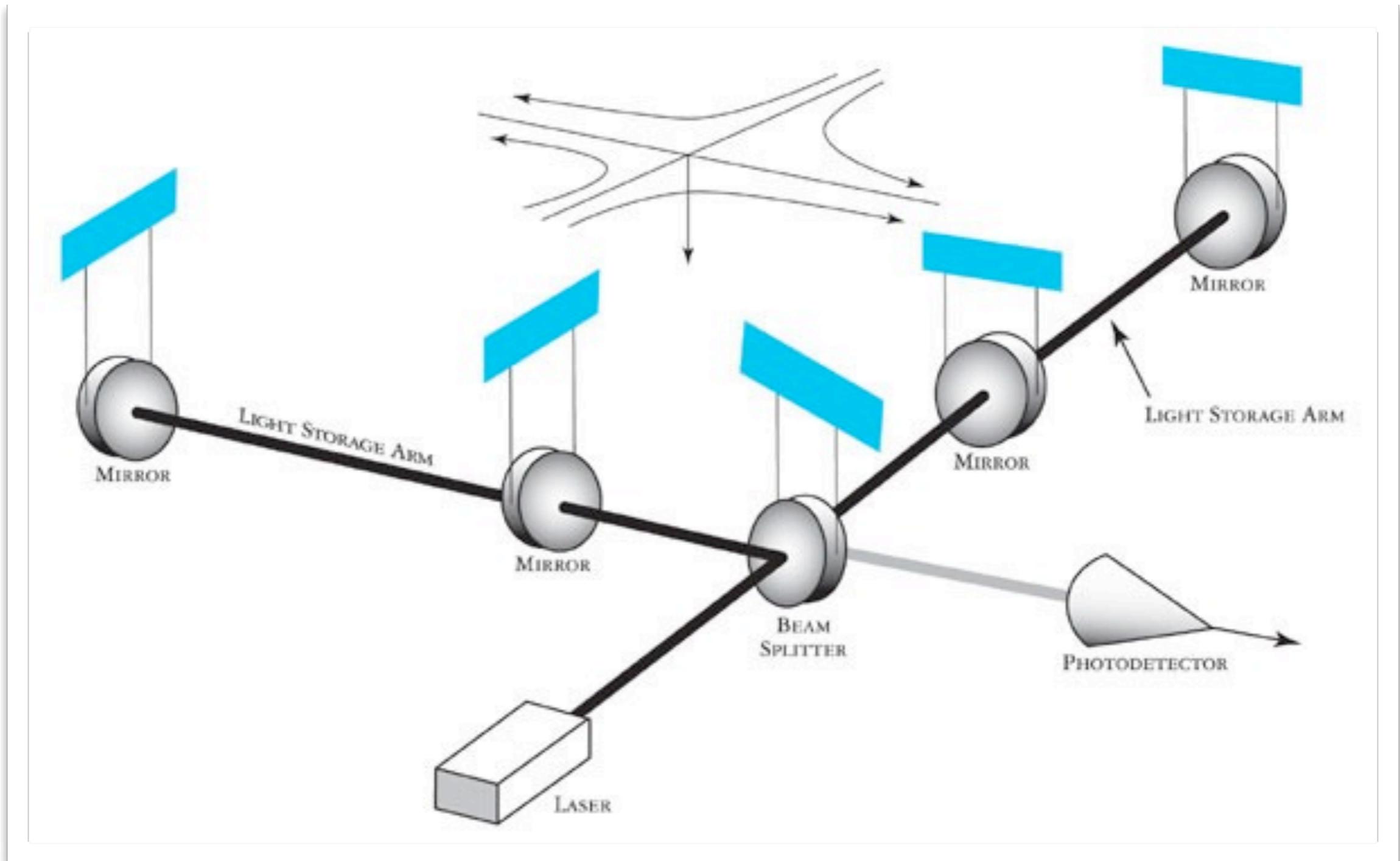


GEO600, near Hannover, Germany
British-German



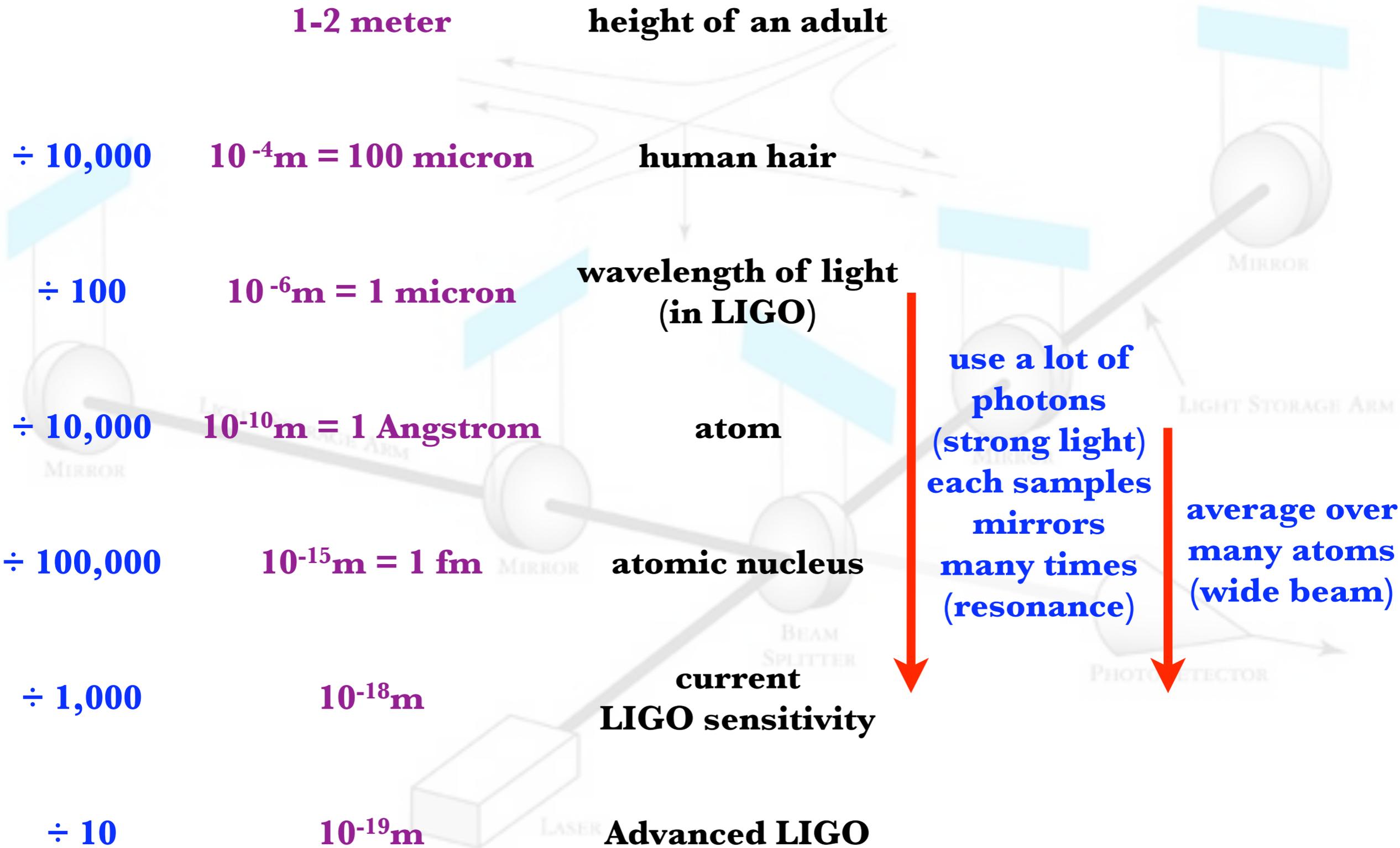
TAMA 300, near Tokyo
Japanese

Michelson Interferometry



Current position sensitivity: 10^{-18} meter = 1 attometer
waves from 40 Hz to 10 kHz

LIGO Sensitivity



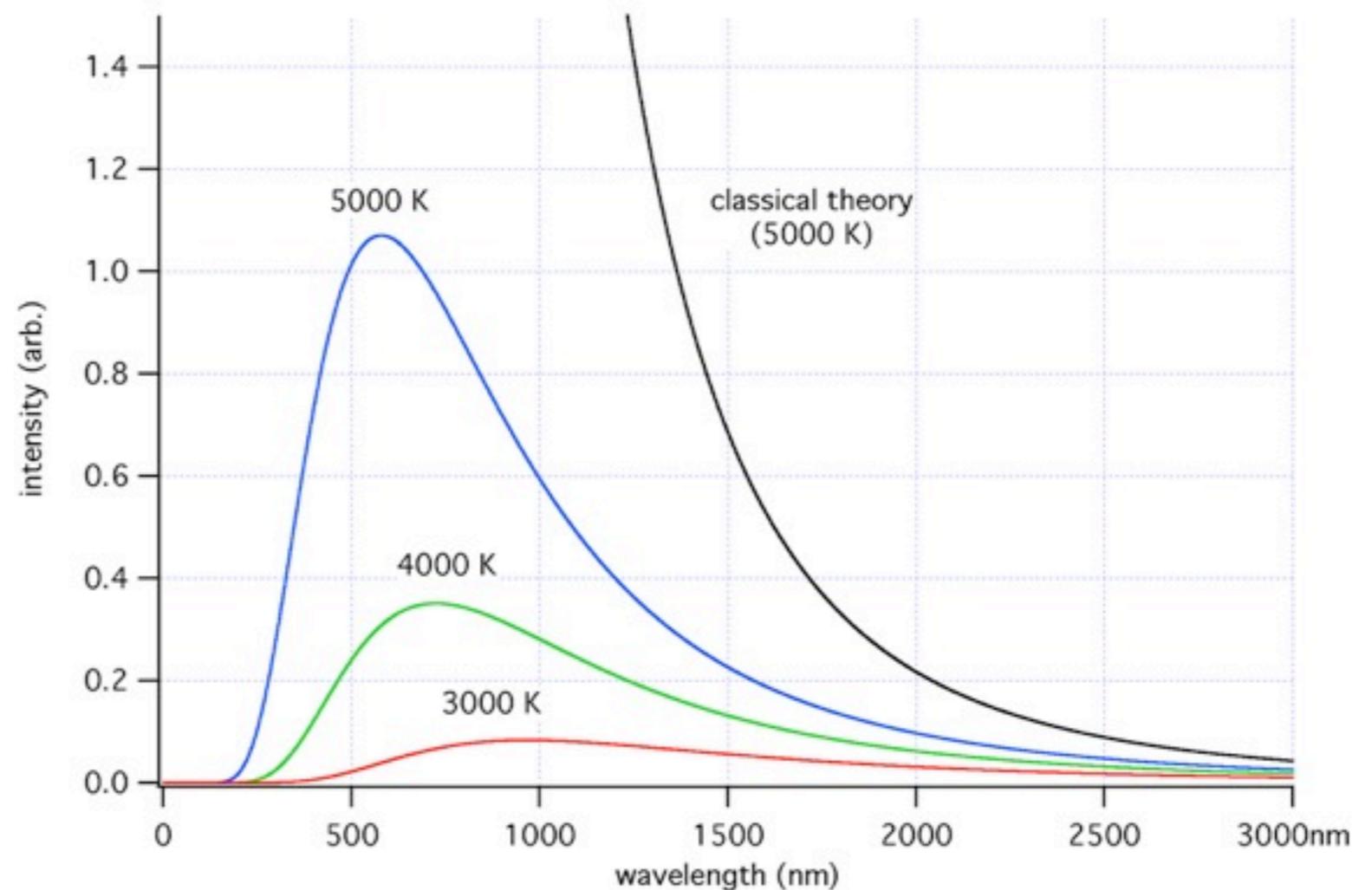
Photons: Black-Body Radiation

- Light energy is *quantized* (broken into pieces, or photons)

$$E = h\nu = \frac{hc}{\lambda}$$



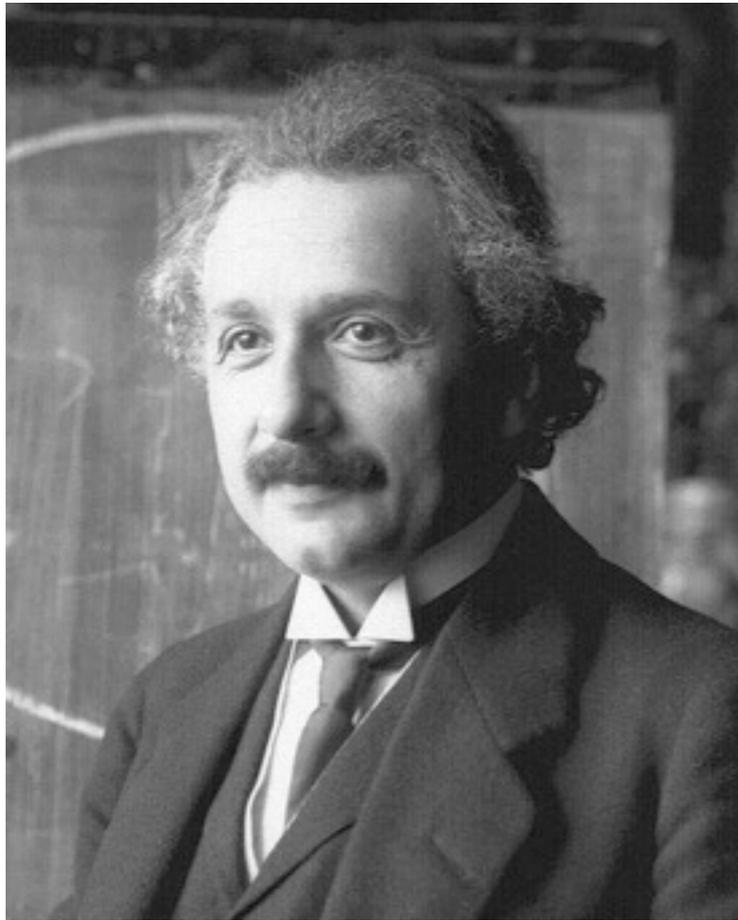
Max Planck
1858-1947



Planck's black-body radiation spectrum (1900)

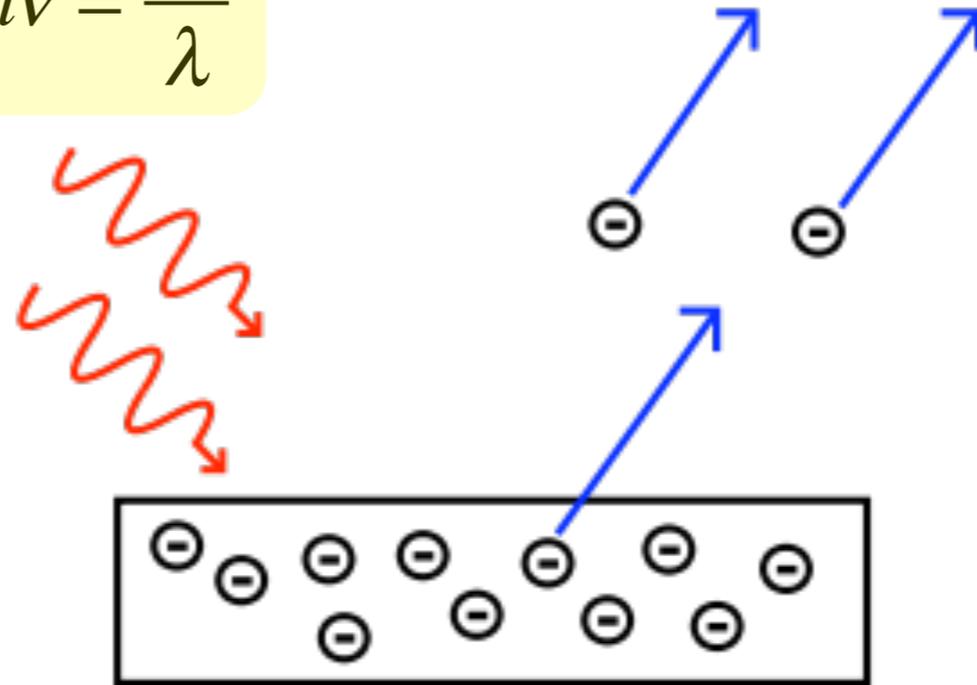
Short-wavelength light not radiated: *thermal energy* not enough to “excite” photons

Photoelectric Effect



Albert Einstein
1879-1955

$$E = h\nu = \frac{hc}{\lambda}$$

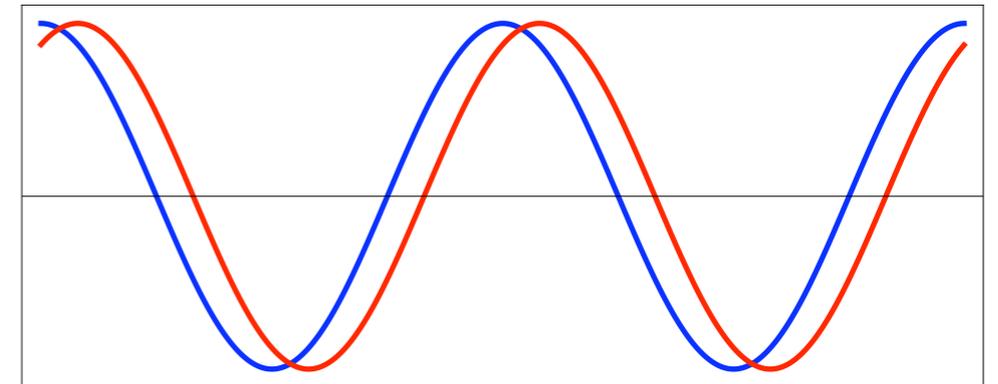
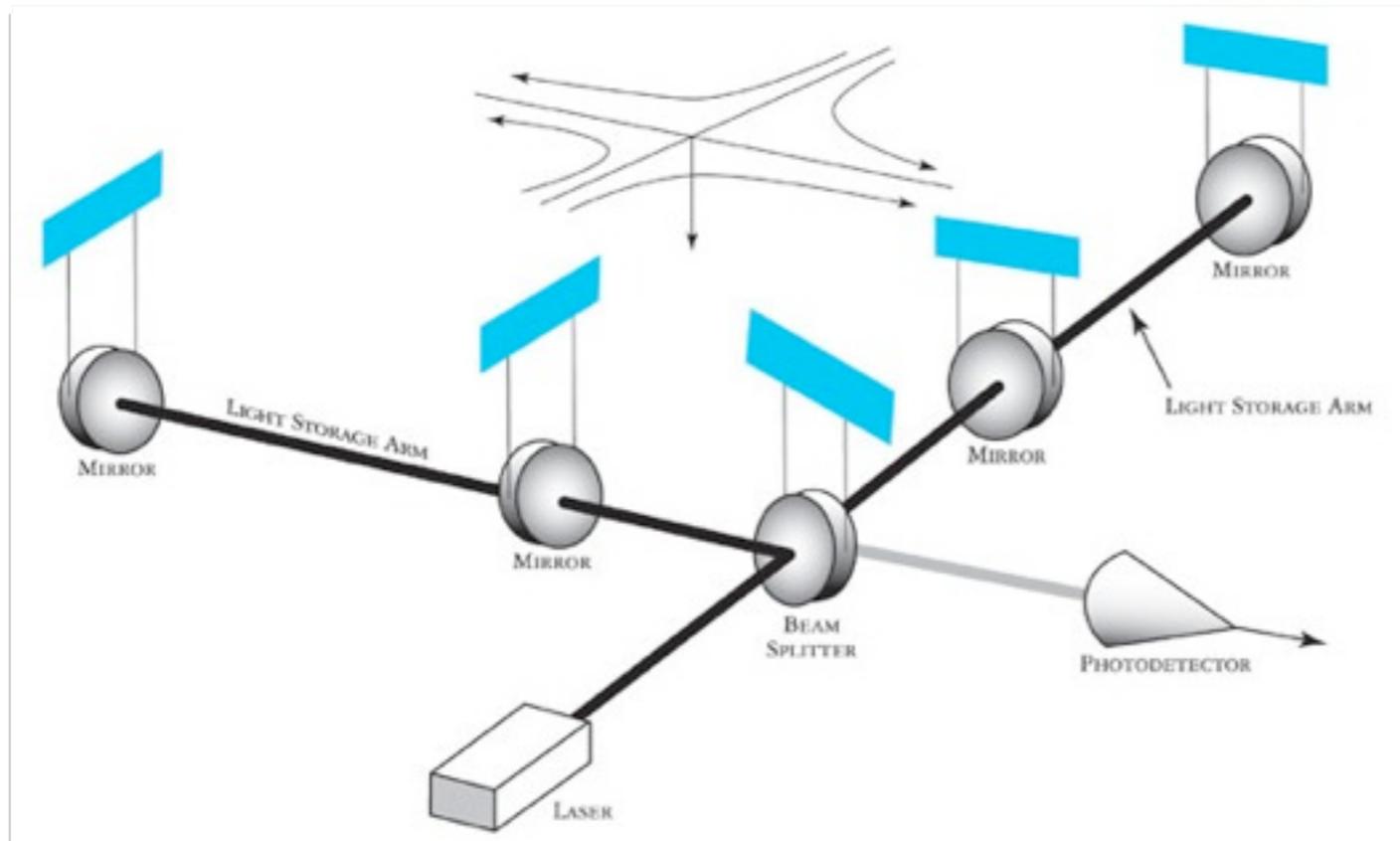


- Only light with short enough wavelength can “knock” electrons out of metal: **because energy delivered *discretely*. (1905)**

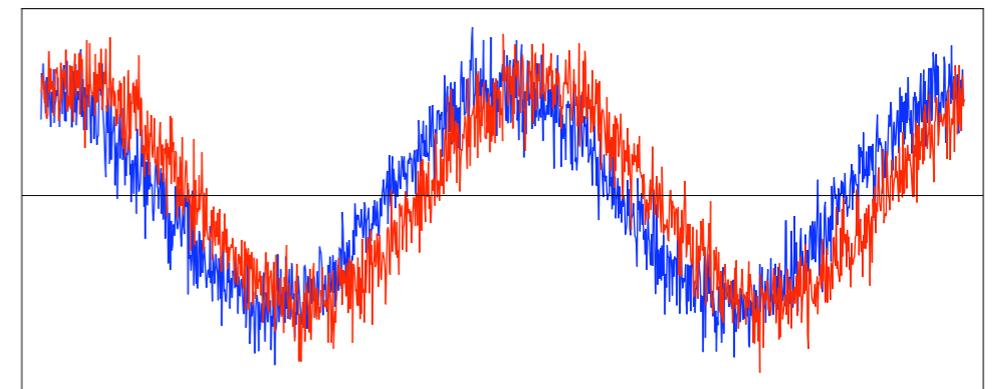
$$h\nu = \phi + E_{kinetic}$$

Photon “shot noise”

- Quantization of light limits measurement accuracy.



classical electromagnetic wave

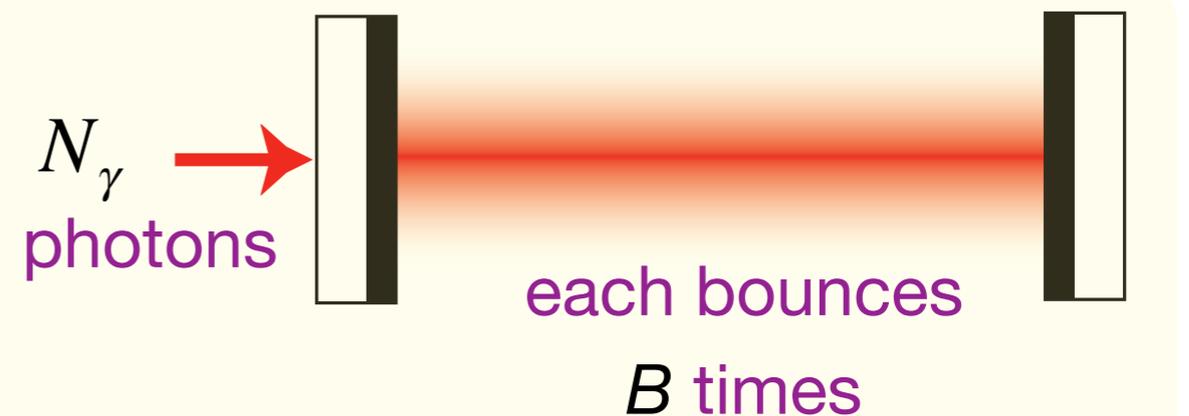


now with quantum fluctuations

- For LIGO, which has Fabry-Perot cavities

$$\delta x \sim \frac{\lambda}{2\pi} \frac{1}{B} \frac{1}{\sqrt{N_\gamma}}$$

- Need to increase # of photons
- ... but this is not yet the whole story*

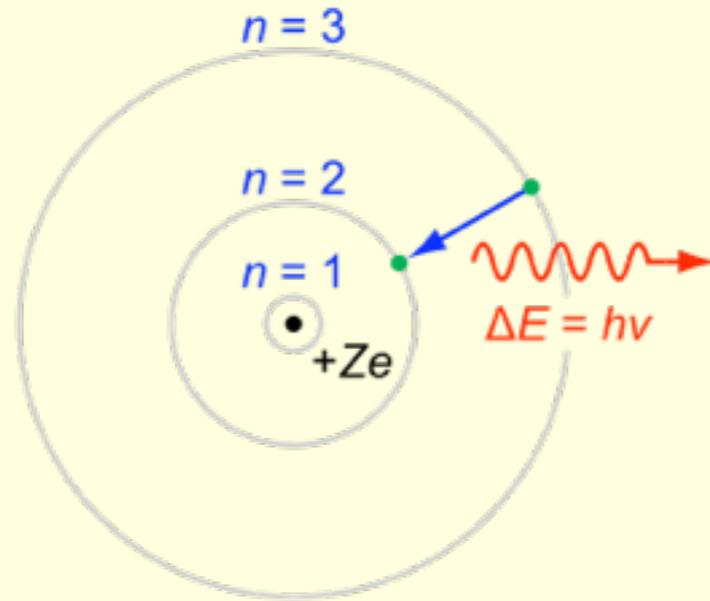


Wave/Particle Duality

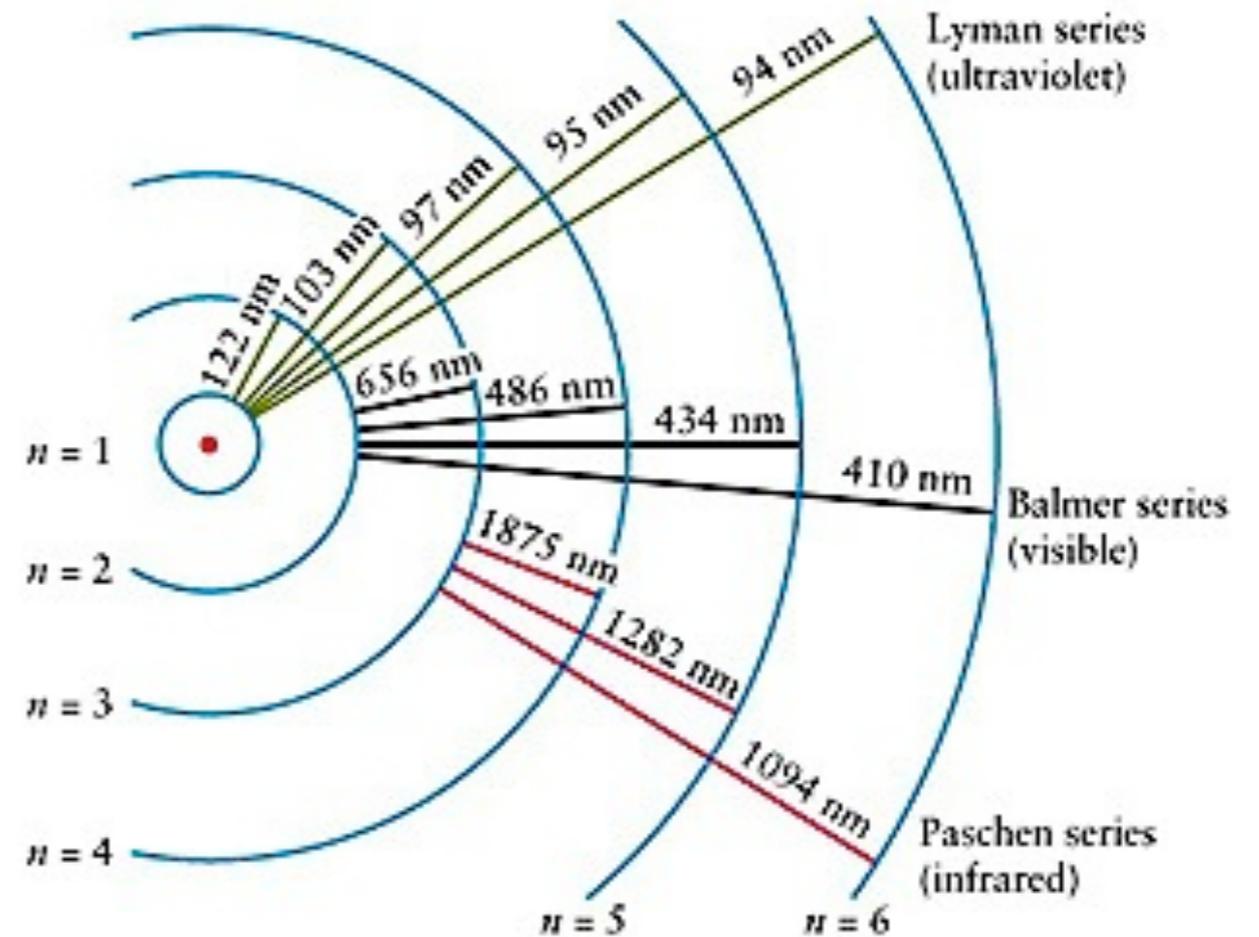
- **Quantization of light:** light is wave --- but also particles
- **Electrons** are particles --- but they are also waves



Niels Bohr



Bohr's model of atom
1913



de Broglie

Electrons are also waves
Bohr's orbits are standing waves

1924

de Broglie
wave length

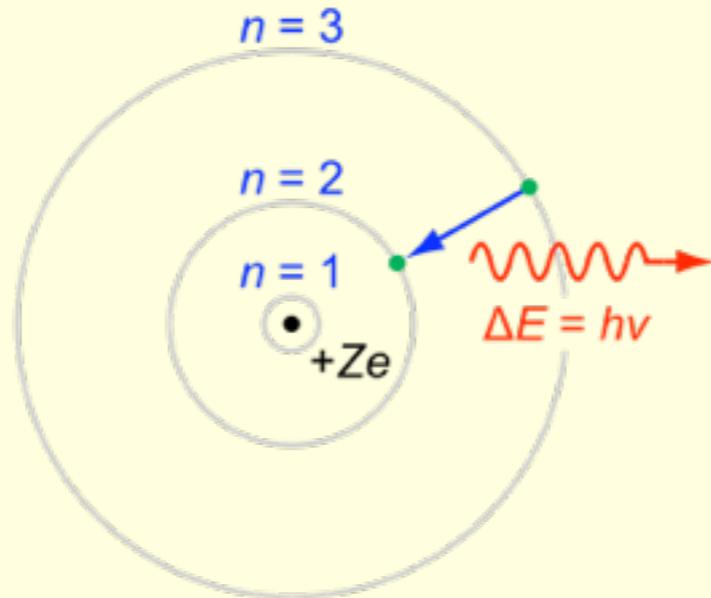
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Wave/Particle Duality

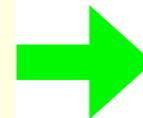
- **Quantization of light:** light is wave --- but also particles
- **Electrons** are particles --- but they are also waves



Niels Bohr



Bohr's model of atom
1913



modern quantum mechanics



Werner Heisenberg



Erwin Schrödinger

The Schrödinger Equation

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}) \Psi(\mathbf{r}, t)$$

Ψ : **wavefunction**

$|\Psi|^2$: **probability density**

de Broglie

Electrons are also waves

Bohr's orbits are standing waves

1924



Quantum Mechanics as Foundation of Modern Physics¹⁶

The Nucleus
(1-10) × 10⁻¹⁵ m

At the center of the atom is a nucleus formed from **nucleons**—protons and neutrons. Each nucleon is made from three **quarks** held together by their strong interactions, which are mediated by gluons. In turn, the nucleus is held together by the **strong** interactions between the gluon and quark constituents of neighboring nucleons. Nuclear physicists often use the exchange of mesons—particles which consist of a quark and an antiquark, such as the **pion**—to describe interactions among the nucleons.

neutron
10⁻¹⁵ m

proton

quark
<10⁻¹⁹ m

strong field

electromagnetic field

In an atom, **electrons** range around the nucleus at distances typically up to 10,000 times the nuclear diameter. If the electron cloud were shown to scale, this chart would cover a small town.

dispersion of massive elements <50 K–3 K >3 × 10⁸ yr today 3 K 14 × 10⁹ yr

density (kg/m³) 0 1 × 10¹⁸ 2 × 10¹⁸

background). Cu provide hints that have glimpsed the gluon plasma.

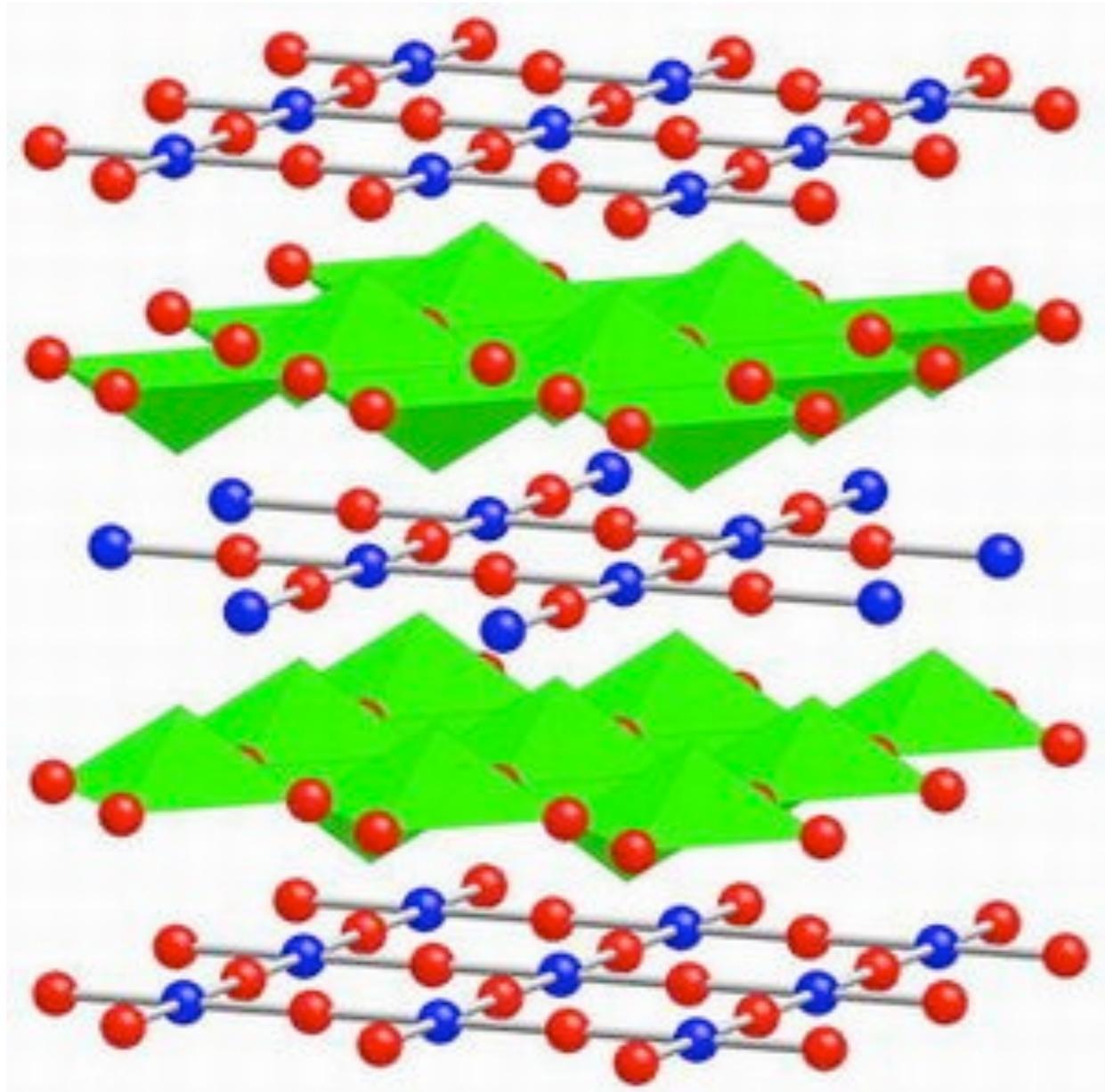
126

50 Sn In Cd Ag Pd Rh Ru Tc Mo Nb

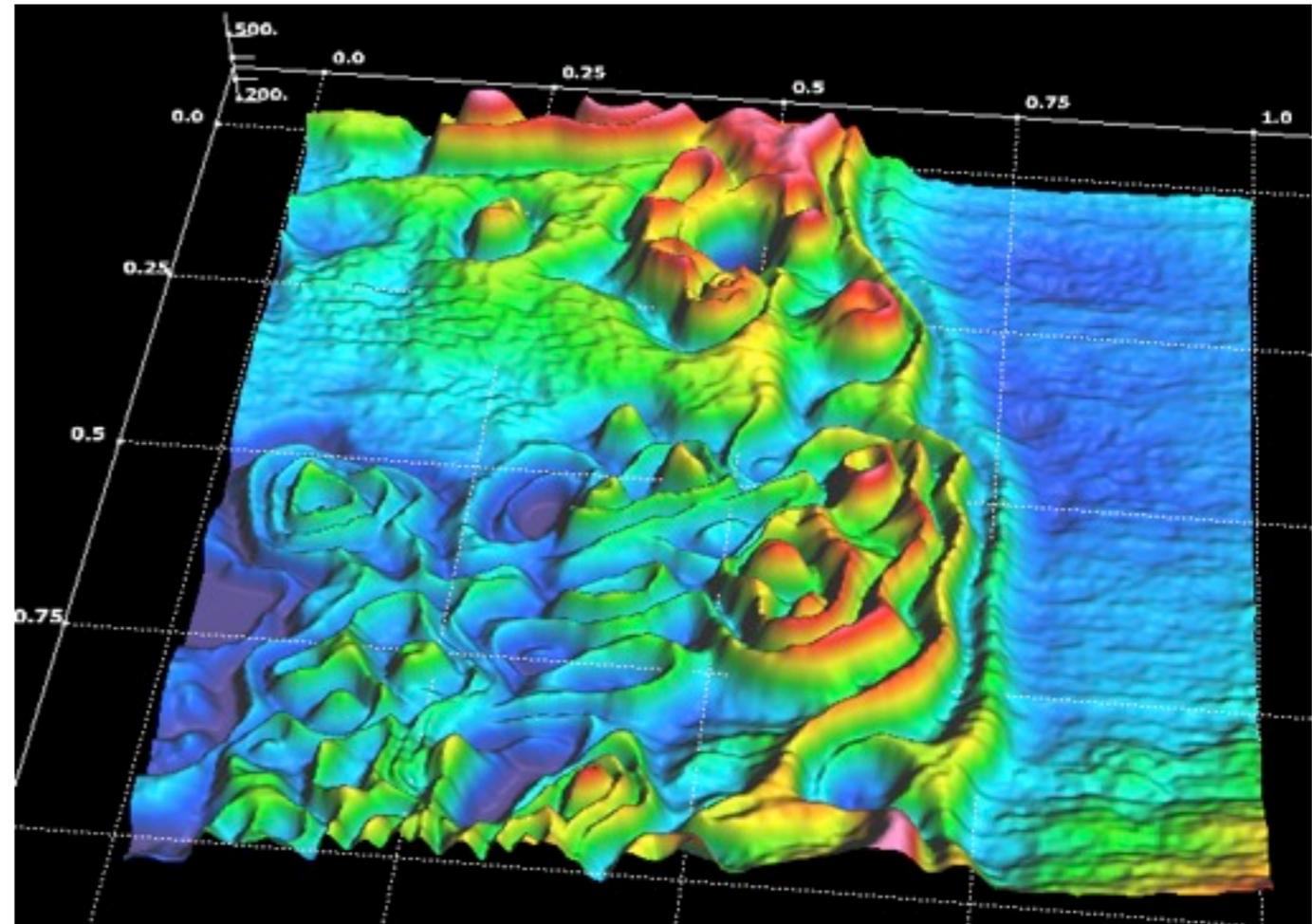
50

Nuclear & Particle Physics: Revealing Deeper Structures of Matter

Quantum Mechanics as Foundation of Modern Physics¹⁷



Structure of superconducting material YBCO
(Argonne National Lab)



Electron density map in a 2-D electron gas
(G. Finkelstein, Duke University)

Condensed Matter Physics: Exotic Properties of Matter

Quantum Mechanics in Modern Technology

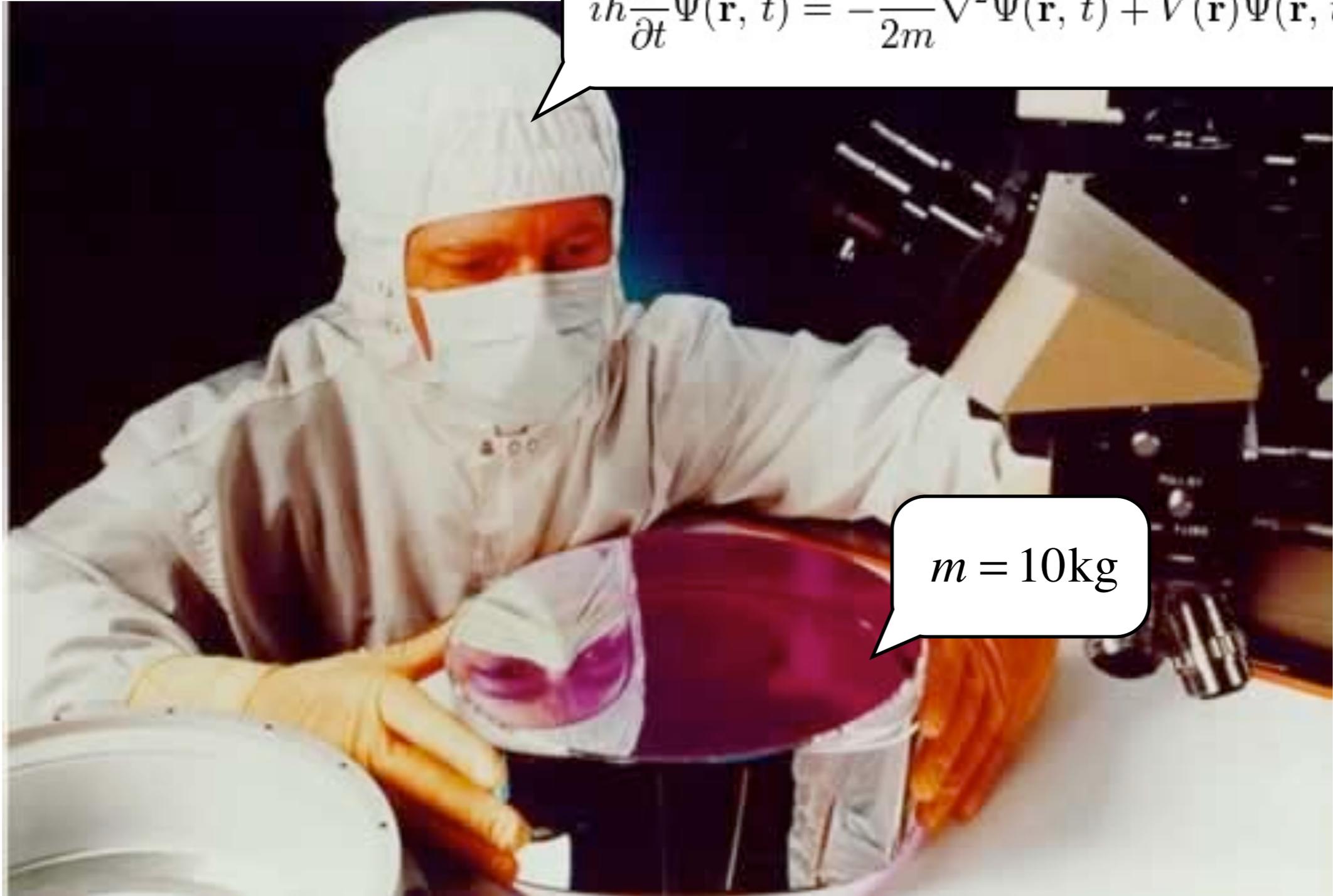


= 40,000 X



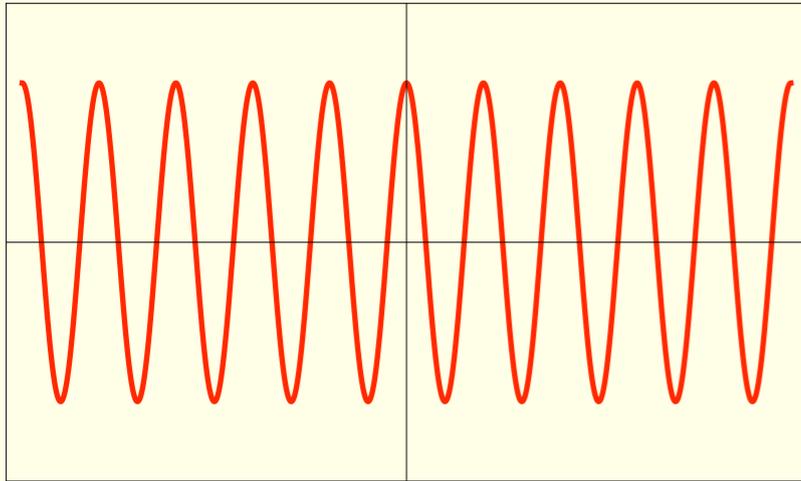
ENIAC: picture from the U of Penn

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}) \Psi(\mathbf{r}, t) ??$$

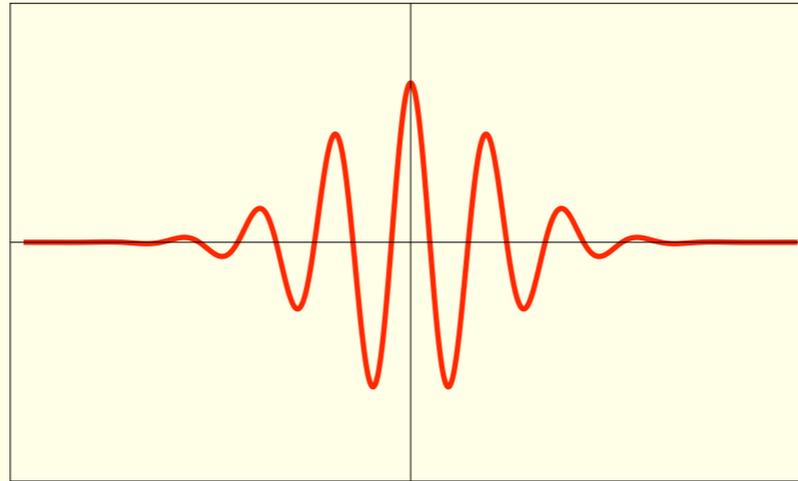


$$m = 10\text{kg}$$

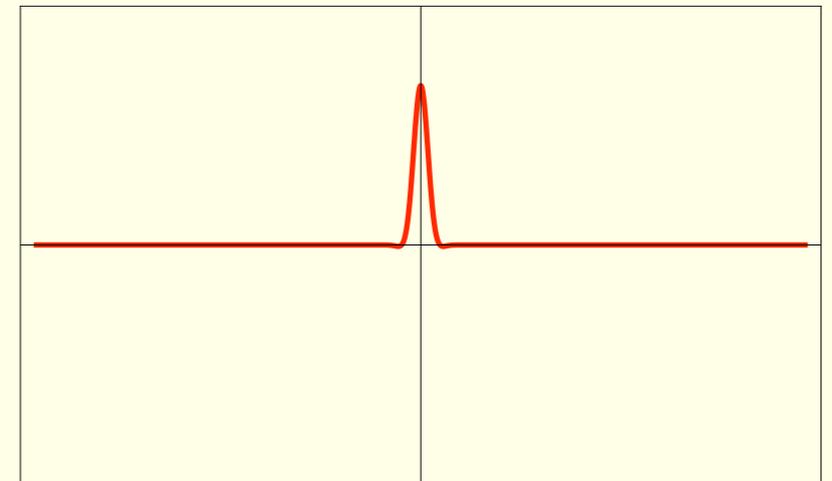
The Heisenberg Uncertainty Principle



a “pure” wave has unique wavelength
cannot be localized at all



a wavy burst contains multiple wavelengths
somewhat localizable



a sharp burst contains many wavelengths
very localizable

Fourier Analysis: $\delta x \times \delta \left(\frac{1}{\lambda} \right) \geq \frac{1}{2\pi}$

- De Broglie Wavelength

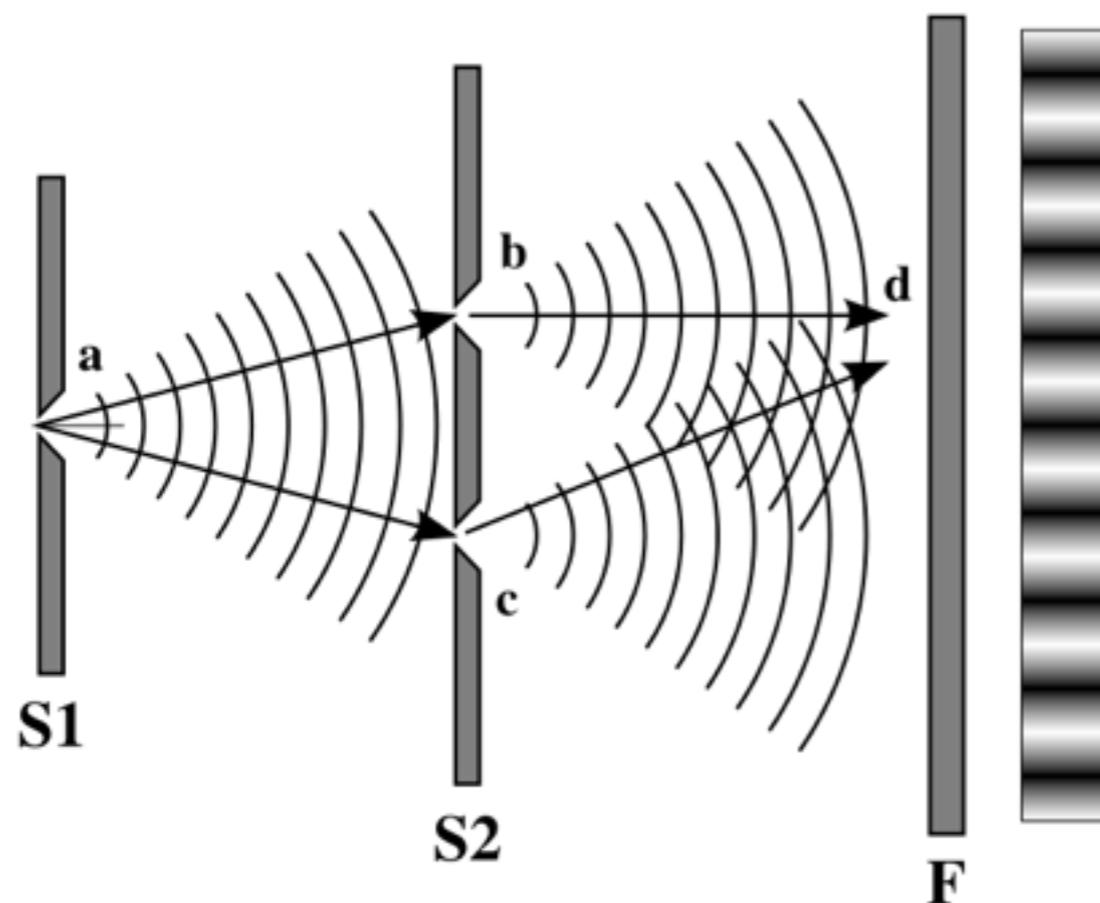
$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \rightarrow \quad p = \frac{h}{\lambda} \quad \rightarrow \quad \delta p \times \delta x \geq \frac{h}{2\pi} \equiv \hbar$$

Position & Momentum (speed) of Particle Cannot be Simultaneously Specified!

Heisenberg Uncertainty Principle

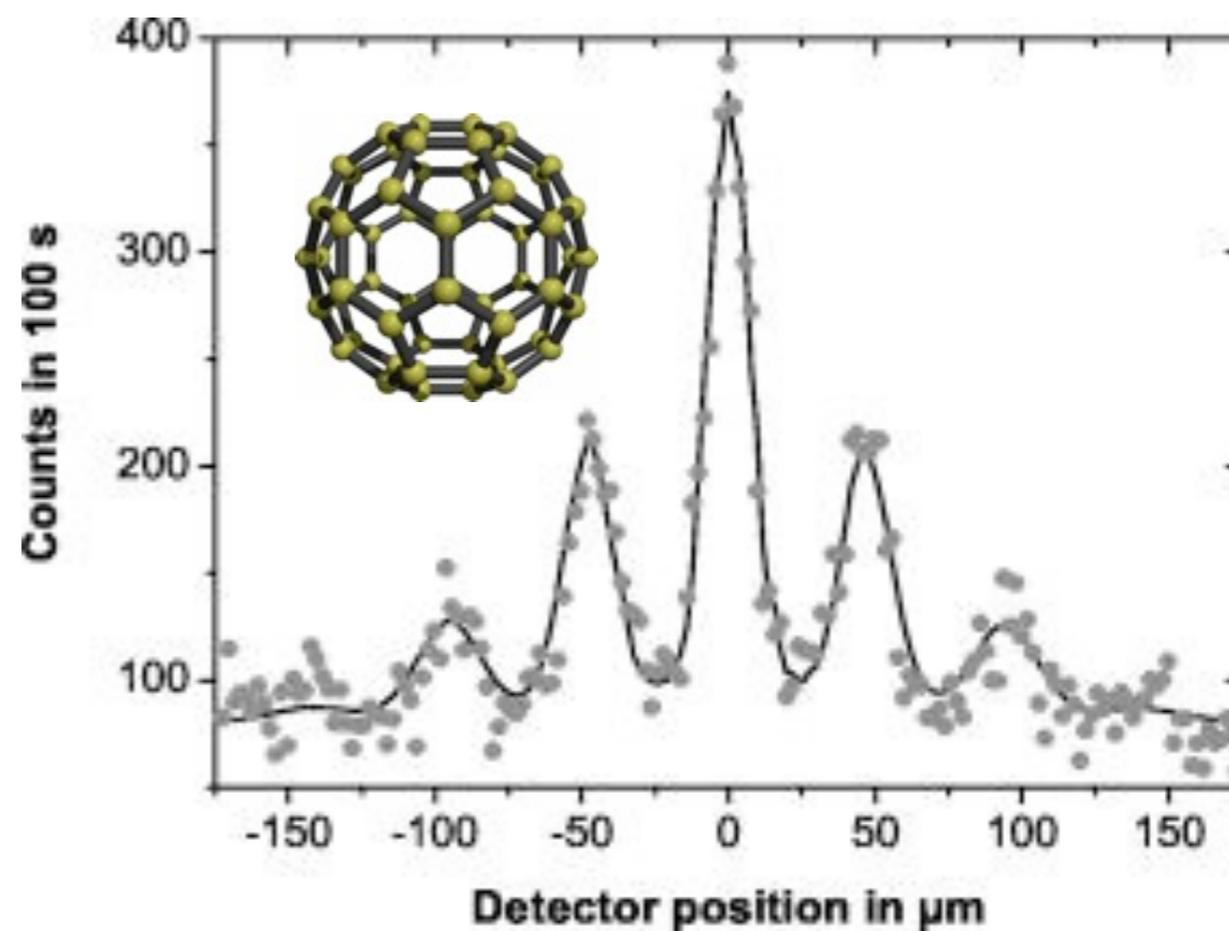
Quantum Superposition

- Waves Interfere: Quantum Superposition



Double Slit for Matter Wave
particle at a superposition state

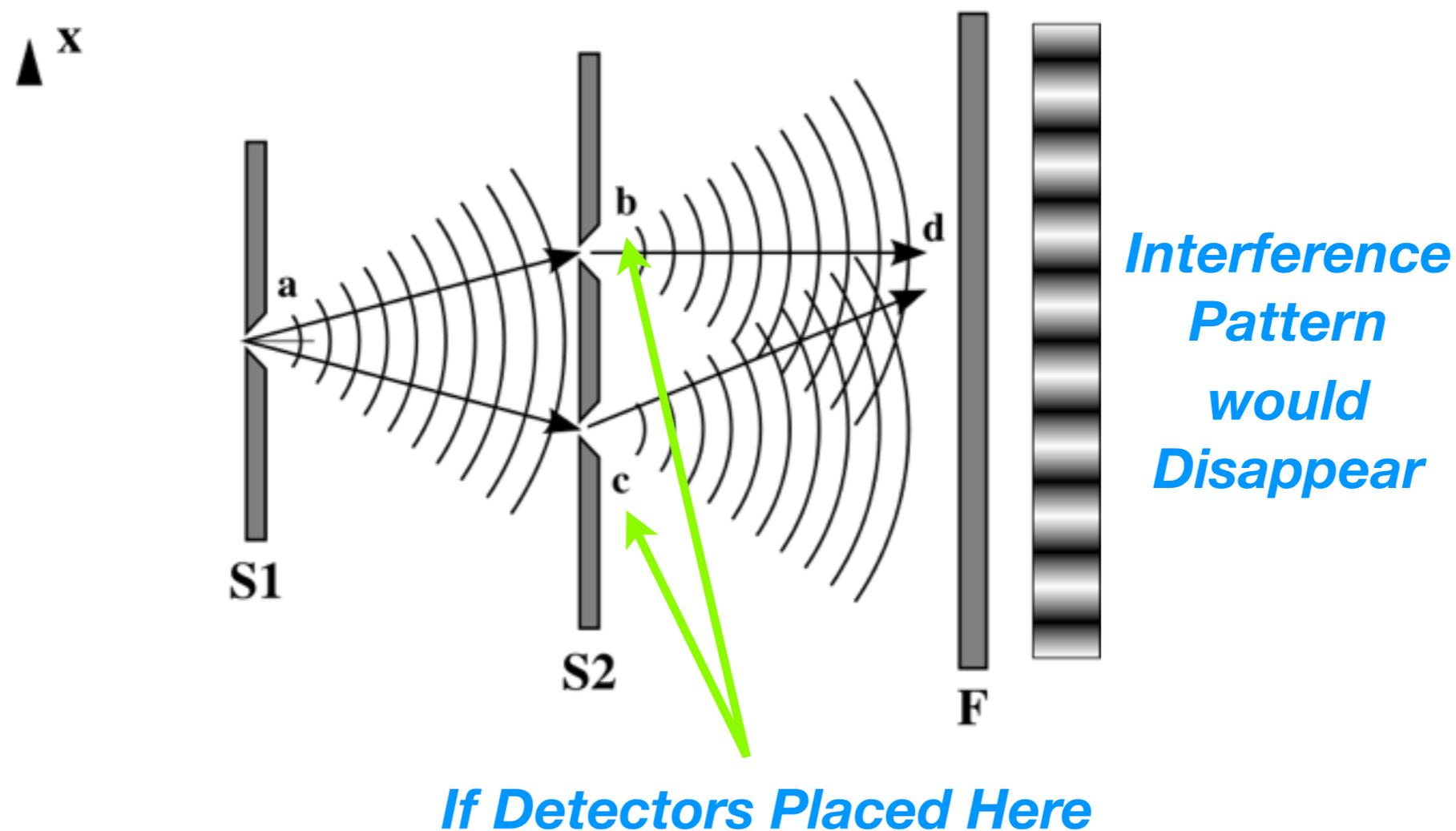
$$|\Psi(S2)\rangle = |b\rangle + |c\rangle$$



Data Using Fullerene Molecule C_{60}
 Research Group of A. Zeilinger in
 Vienna

Collapse of Wave Function due to Measurement

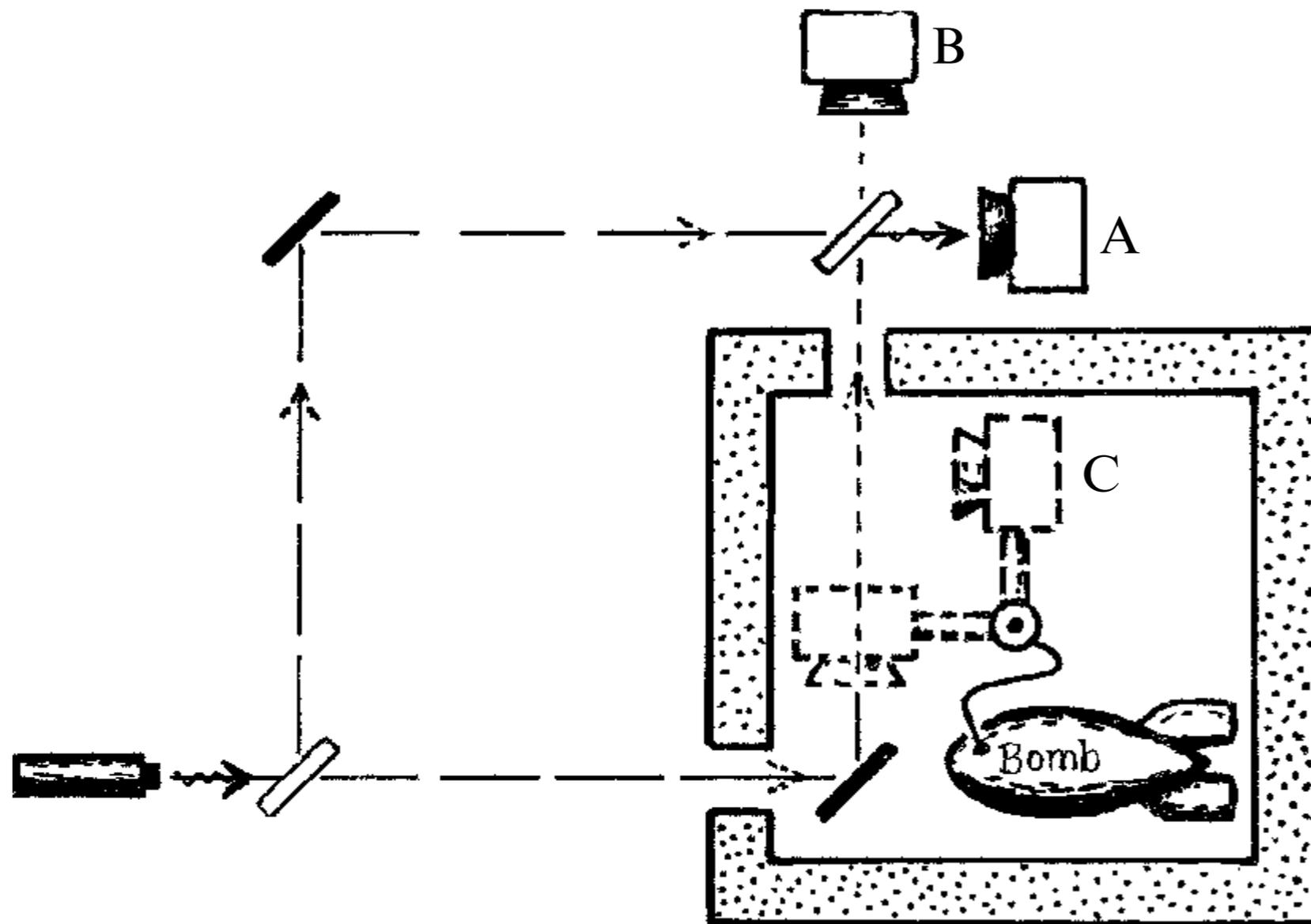
- Measurement Collapses Wave Function
- Can destroy interference pattern (*loss of quantum coherence, or decoherence*)



$$|\Psi(S2)\rangle = |b\rangle + |c\rangle \quad \longrightarrow \quad |b\rangle \text{ or } |c\rangle$$

quantum superposition
measurement
classical choice

Bomb Testing “Experiment”



Bad bomb: mirror fixed, photon always appear in **A** port

Good bomb: mirror movable, *measures* photon, so **50%** chance for photon to appear in **B**

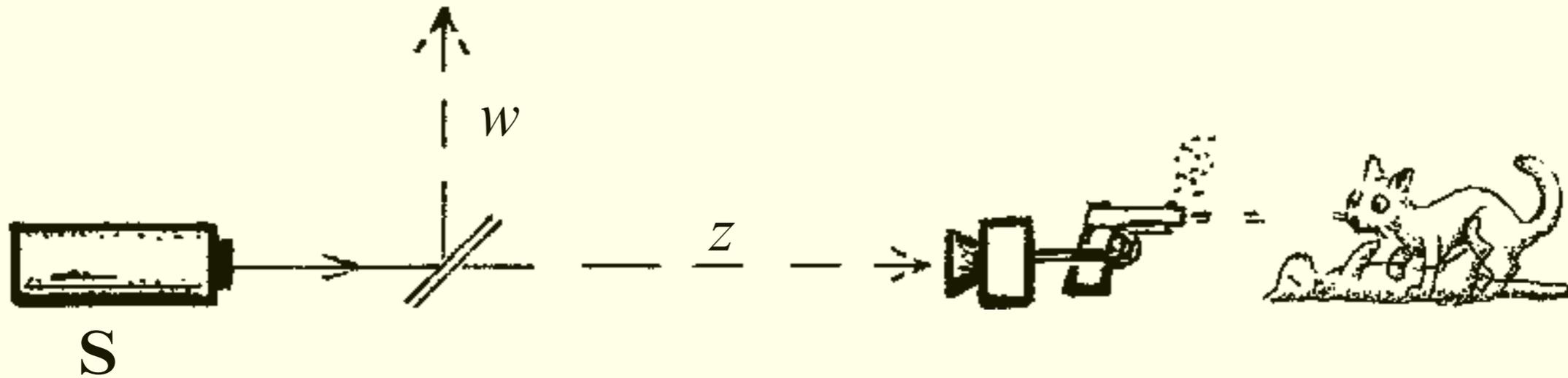
Yet: photon appearing in B doesn't mean it has gone through the path with bomb

Elitzur-Vaidman Bomb Test (Drawing by Roger Penrose)

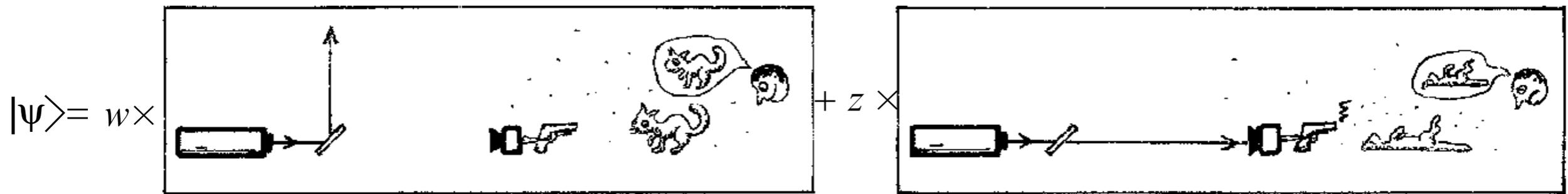
How do we make sure a bomb is good without detonation

Tested by Zeilinger et al. (not with bombs)

Macroscopic quantum superpositions?



Schrödinger's cat thought experiment (picture by Roger Penrose)

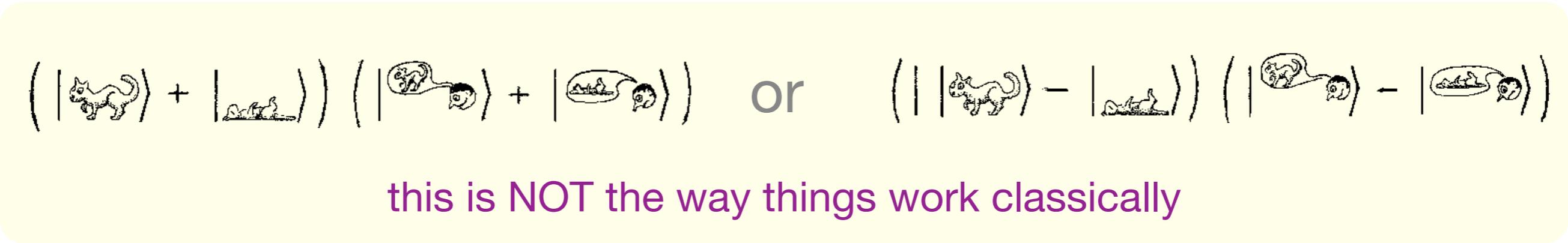
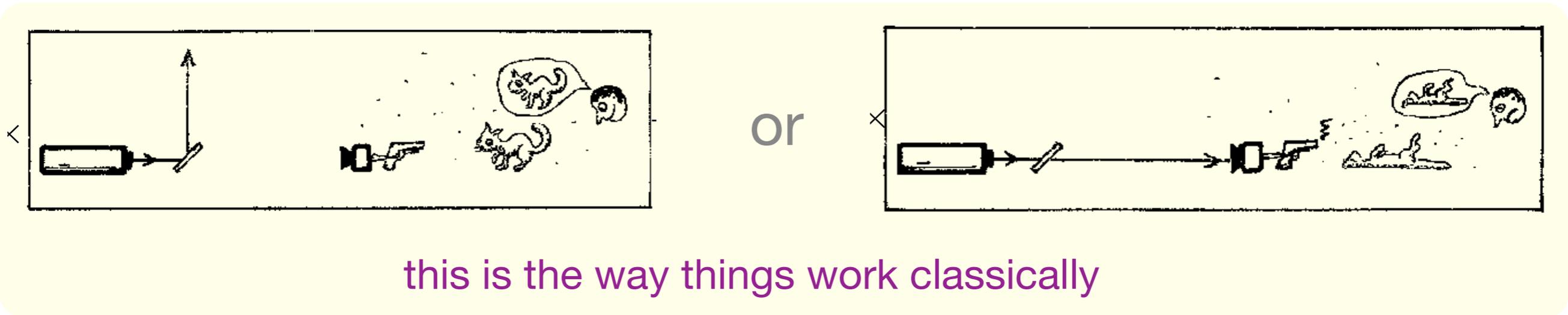


Mathematically Equivalent to



$$2 |\psi\rangle = \left(\left| \begin{array}{c} \text{cat alive} \\ \text{cat dead} \end{array} \right\rangle + \left| \begin{array}{c} \text{cat dead} \\ \text{cat alive} \end{array} \right\rangle \right) \left(\left| \begin{array}{c} \text{observer says meow} \\ \text{observer says poor} \end{array} \right\rangle + \left| \begin{array}{c} \text{observer says poor} \\ \text{observer says meow} \end{array} \right\rangle \right) \\ + \left(\left| \begin{array}{c} \text{cat alive} \\ \text{cat dead} \end{array} \right\rangle - \left| \begin{array}{c} \text{cat dead} \\ \text{cat alive} \end{array} \right\rangle \right) \left(\left| \begin{array}{c} \text{observer says meow} \\ \text{observer says poor} \end{array} \right\rangle - \left| \begin{array}{c} \text{observer says poor} \\ \text{observer says meow} \end{array} \right\rangle \right)$$

How does Quantum transition into Classical?



Question:

Why is $| \text{cat} \rangle$ vs. $| \text{no cat} \rangle$ classical ?

instead of $(| \text{cat} \rangle + | \text{no cat} \rangle)$ vs. $(| | \text{cat} \rangle - | \text{no cat} \rangle)$?

What determines the choice? How is it implemented?

Different Thoughts on Quantum Classical Transition 26

Why is $|\text{cat standing}\rangle$ vs. $|\text{cat sitting}\rangle$ classical ?

instead of $(|\text{cat standing}\rangle + |\text{cat sitting}\rangle)$ vs. $(|\text{cat standing}\rangle - |\text{cat sitting}\rangle)$?

What determines the choice? How is it implemented?

Prevalent answer:

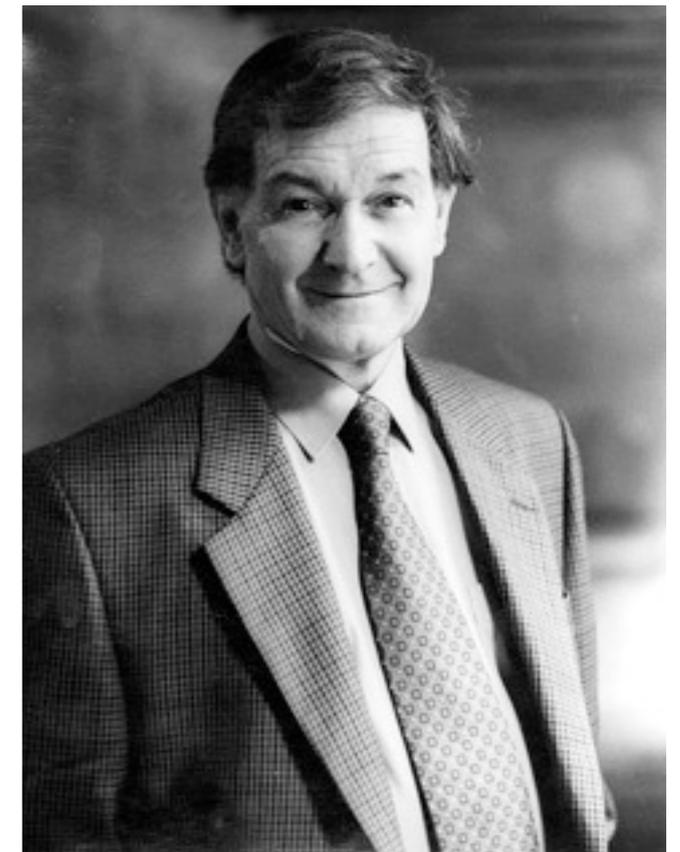
macroscopic systems are in constant contact with the “environment”

environment **measures** the system, and collapses it into **classical states**.

(Environmental Decoherence)

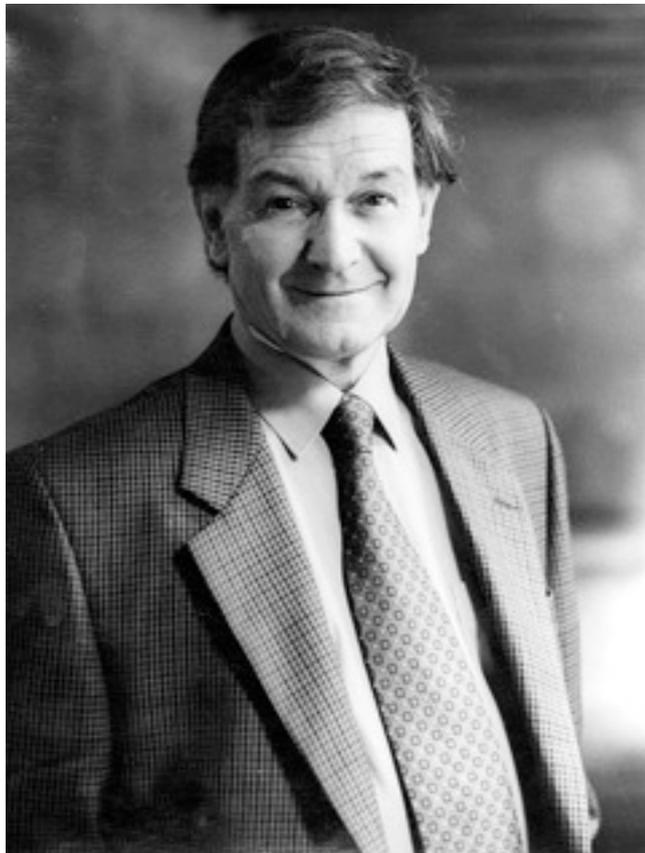
Environment influences the decision of **which states are classical**

Enough isolation with environment prevents classical physics from emerging



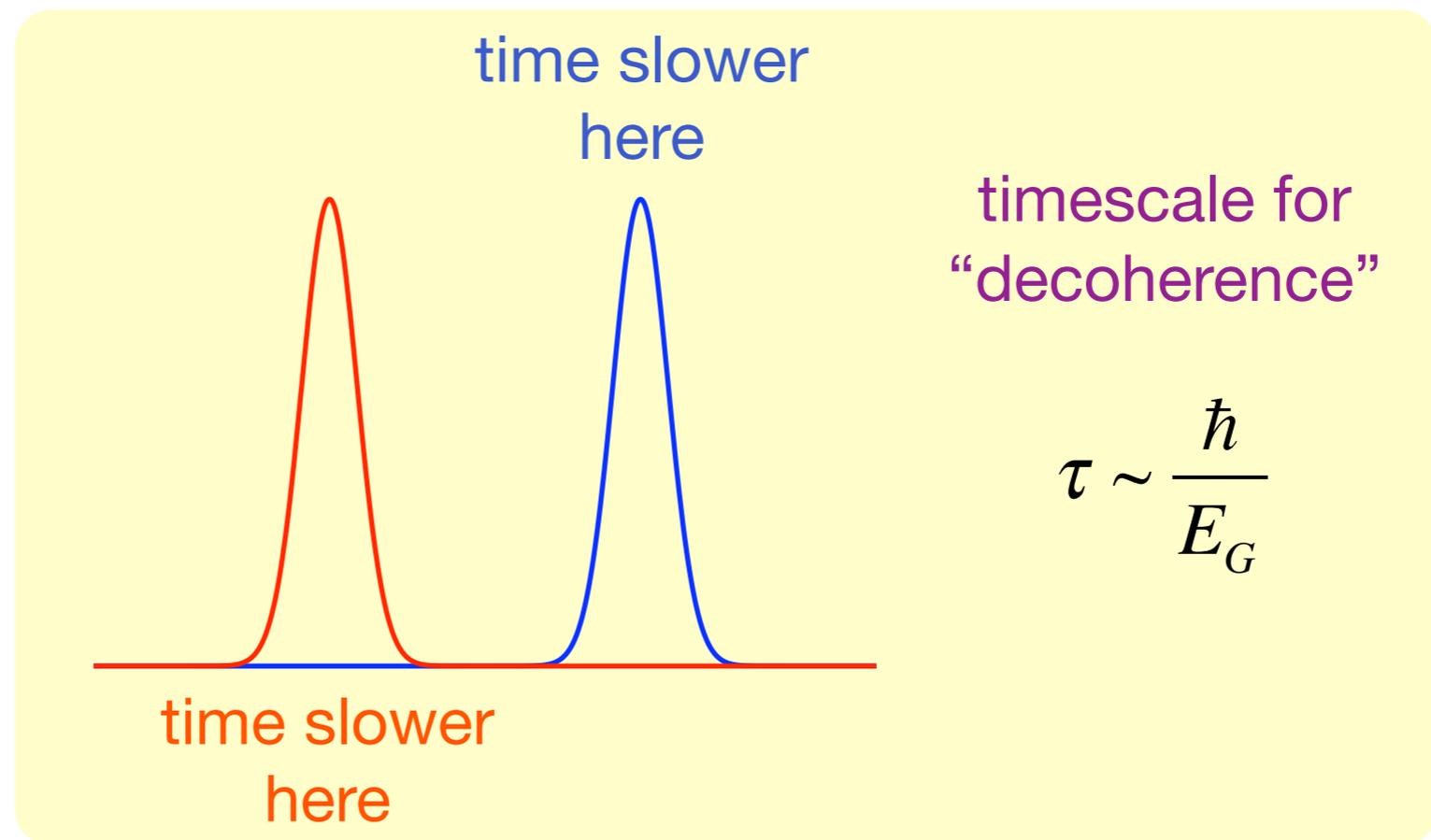
- **Roger Penrose:** quantum superposition will be destroyed by gravity. **“Gravity Decoherence”**

Gravity Decoherence



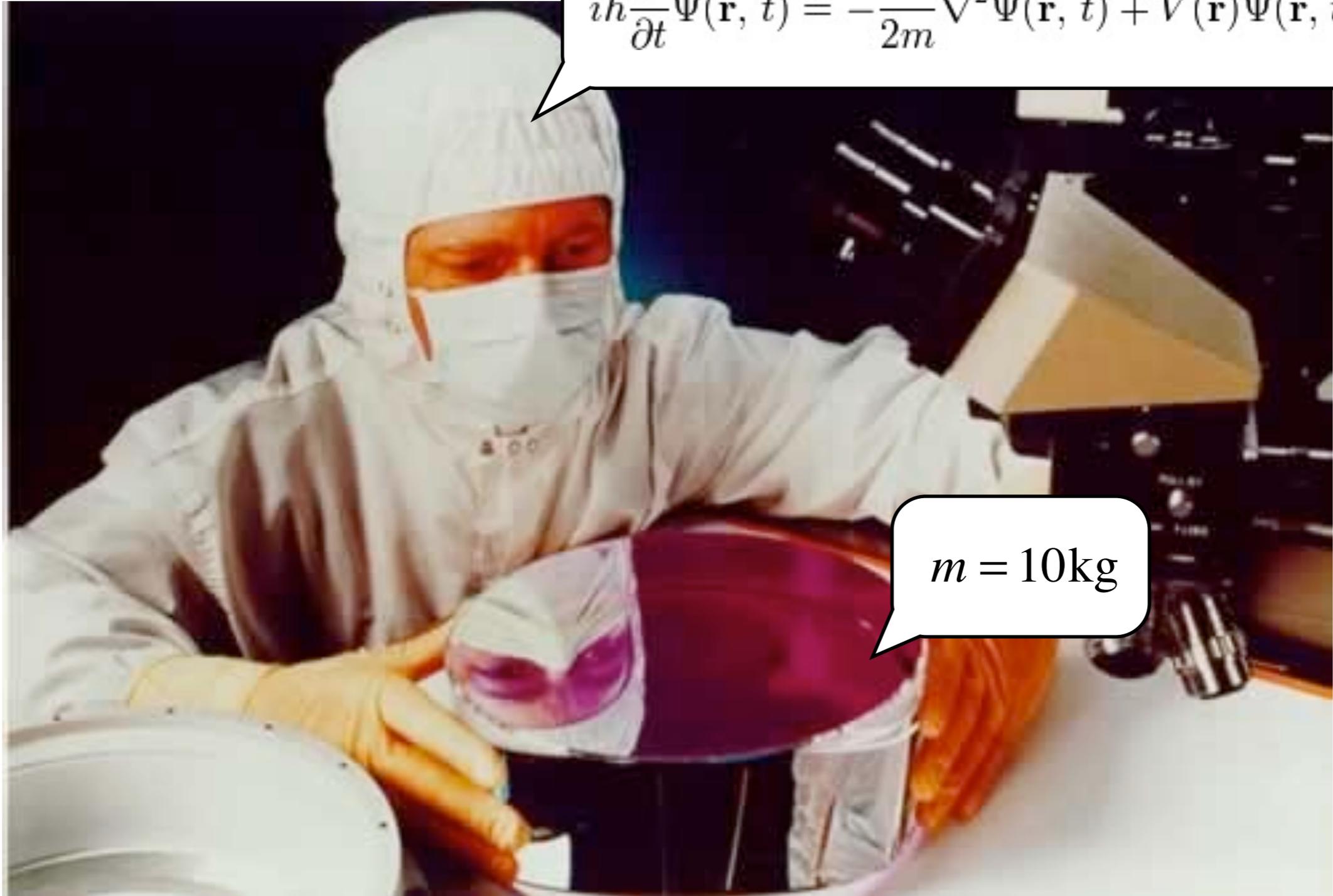
Sir Roger Penrose

- Roger Penrose: “Gravity Decoherence”
- Motivation:
 - quantum superposition, through gravity, cause superposition in space-time structure, which **must disappear** quickly



- Further conjectured that **consciousness must be quantum**

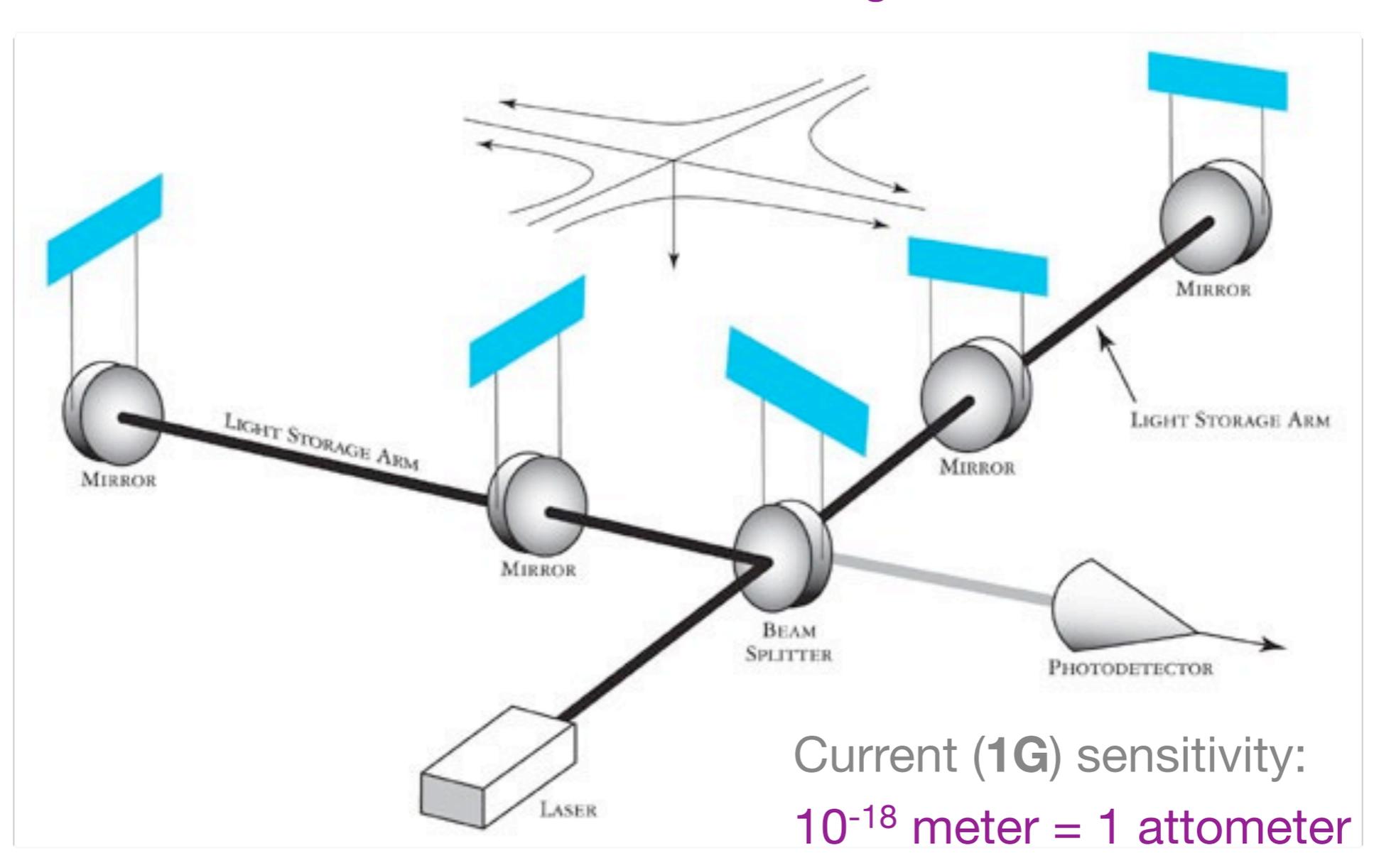
$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}) \Psi(\mathbf{r}, t) ??$$



$$m = 10\text{kg}$$

How does Quantum Mechanics Affect LIGO

- If Quantum Mechanics works in LIGO, then 10 kg test masses are also like waves



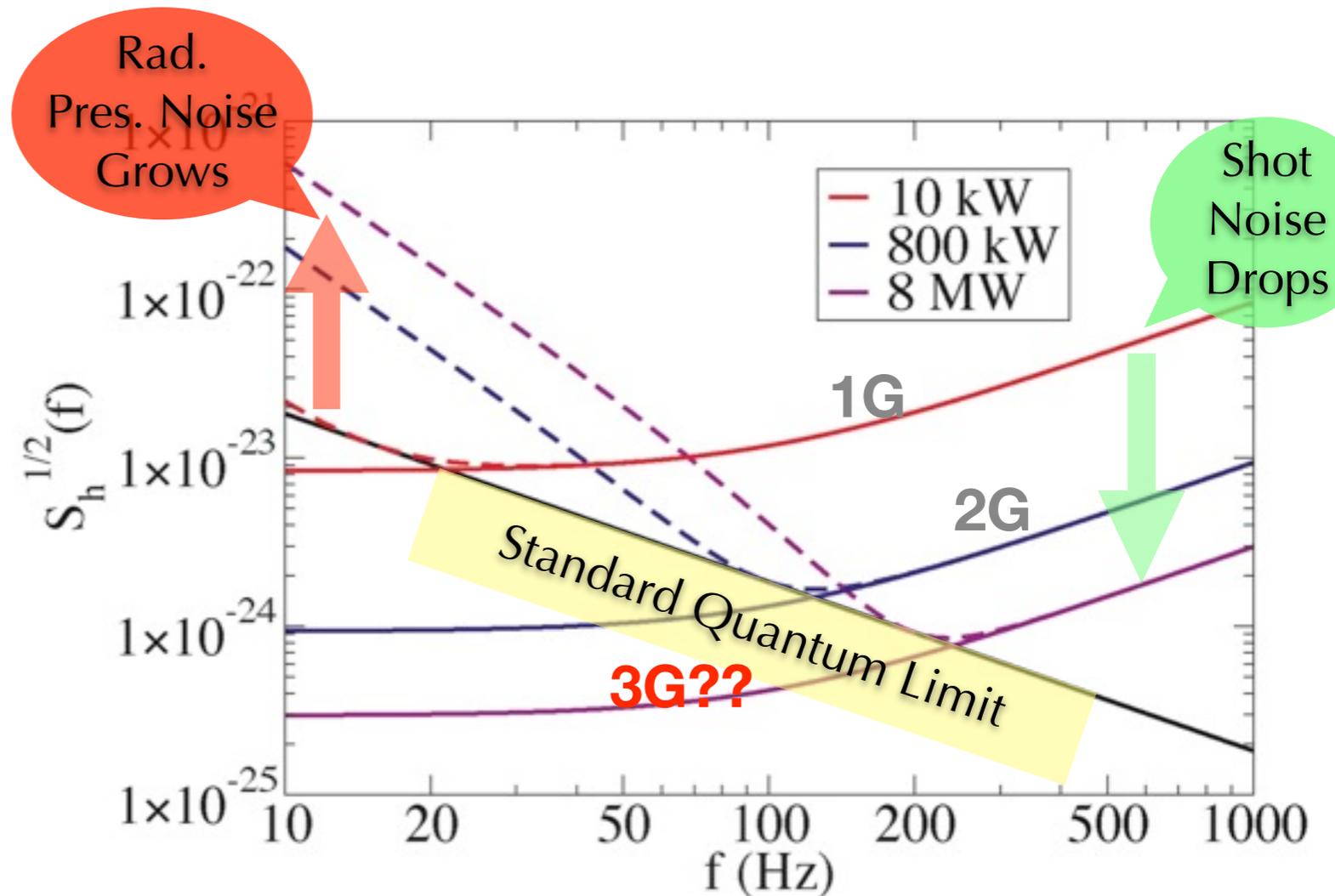
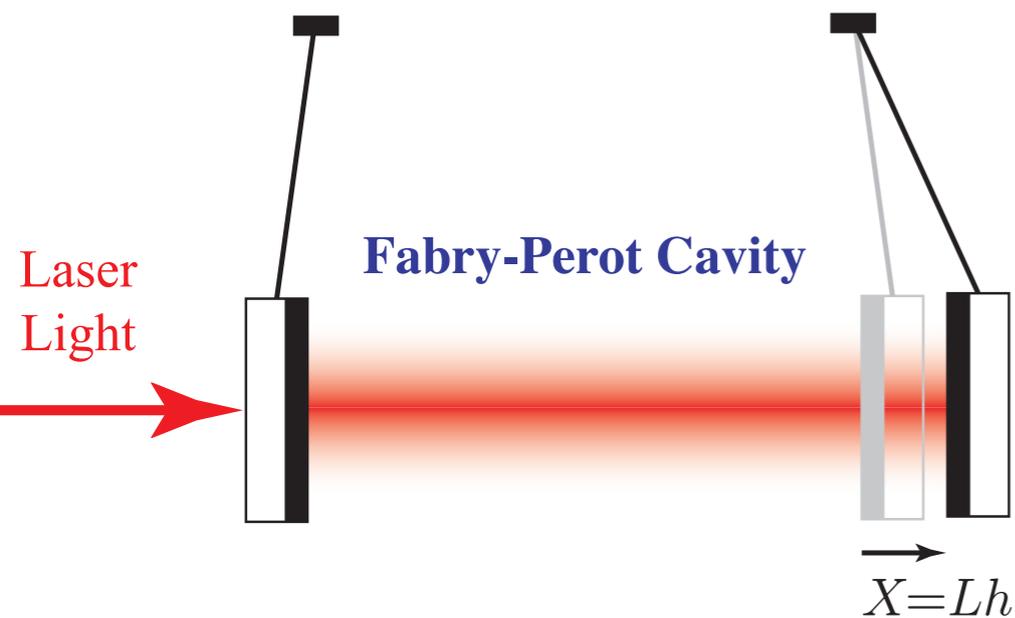
$$\delta x \cdot \delta p \sim \hbar \sim 10^{-34} \quad \delta p \sim m \cdot \delta v \sim m\omega \cdot \delta x$$

$$\delta x \sim \sqrt{\frac{\hbar}{m\omega}} \sim 10^{-19} \text{ m, at } 100\text{Hz}$$

Only 10 from Heisenberg Uncertainty!!
 Advanced LIGO (2G)
 (already started construction)
 has 10x sensitivity!!

Symptom of Heisenberg Uncertainty

- Shot noise decrease when we increase photon number.
- But photons also kick the mirrors randomly. This effect increase with photon number



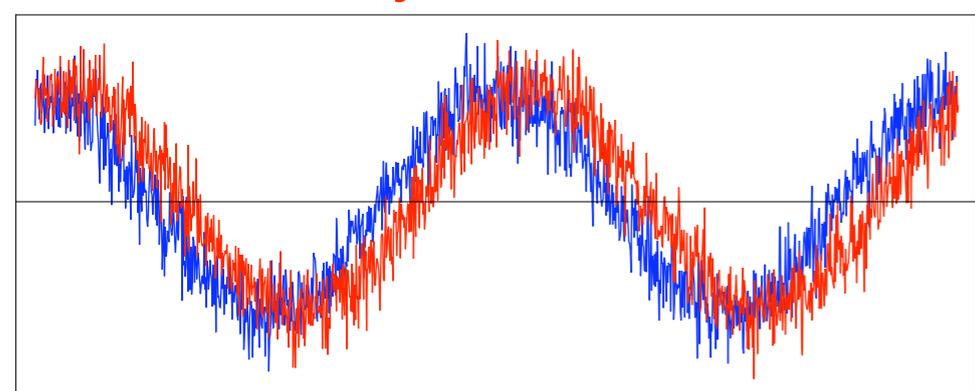
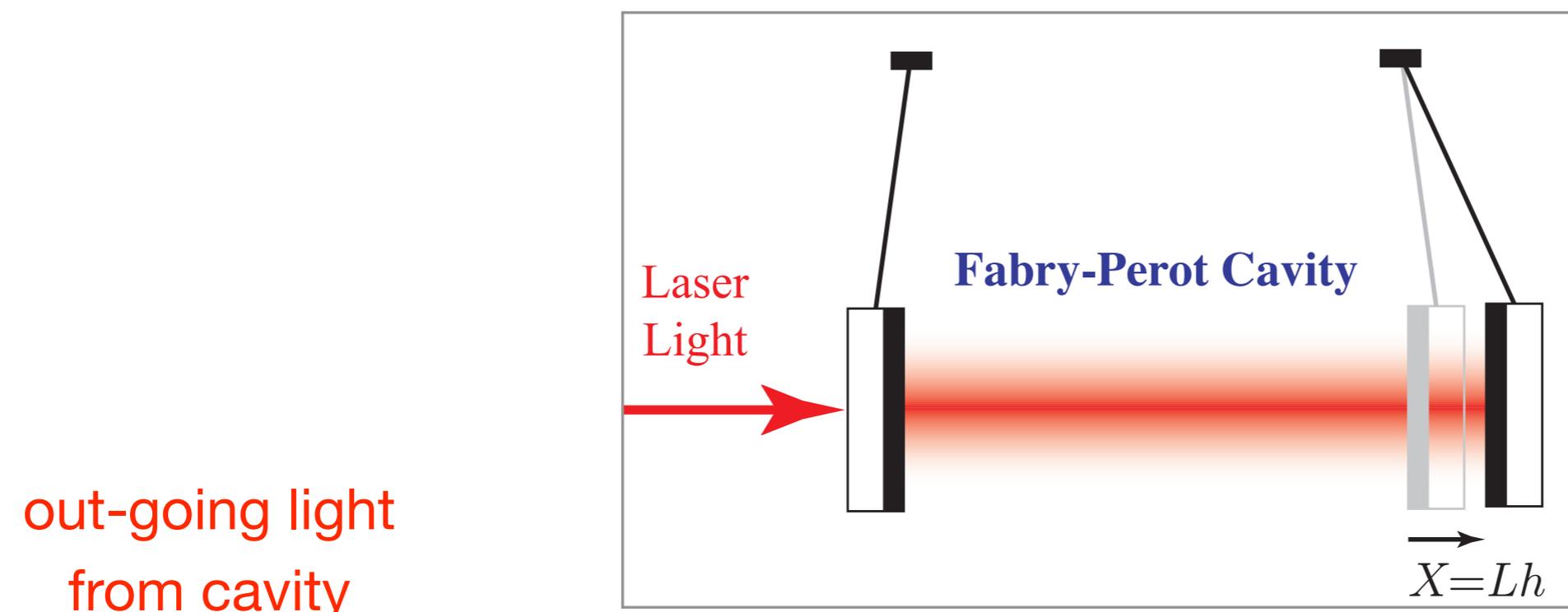
The Standard Quantum Limit poses challenge toward further improvement

$$\delta x \sim \sqrt{\frac{\hbar}{m\omega}}$$

could use heavy mirrors, but not very efficient

How may we circumvent the Quantum Limit?

- Coherent removal of radiation-pressure noise



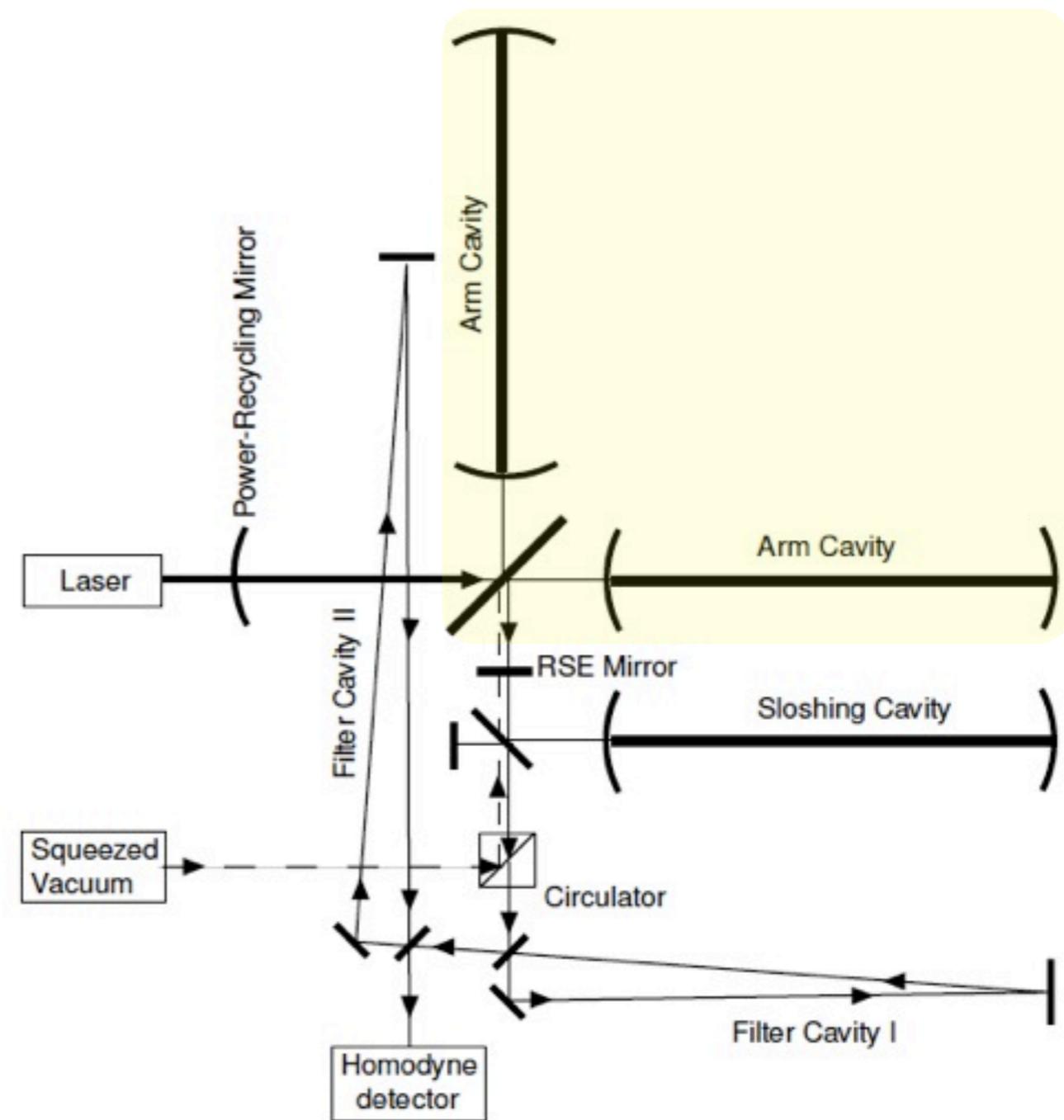
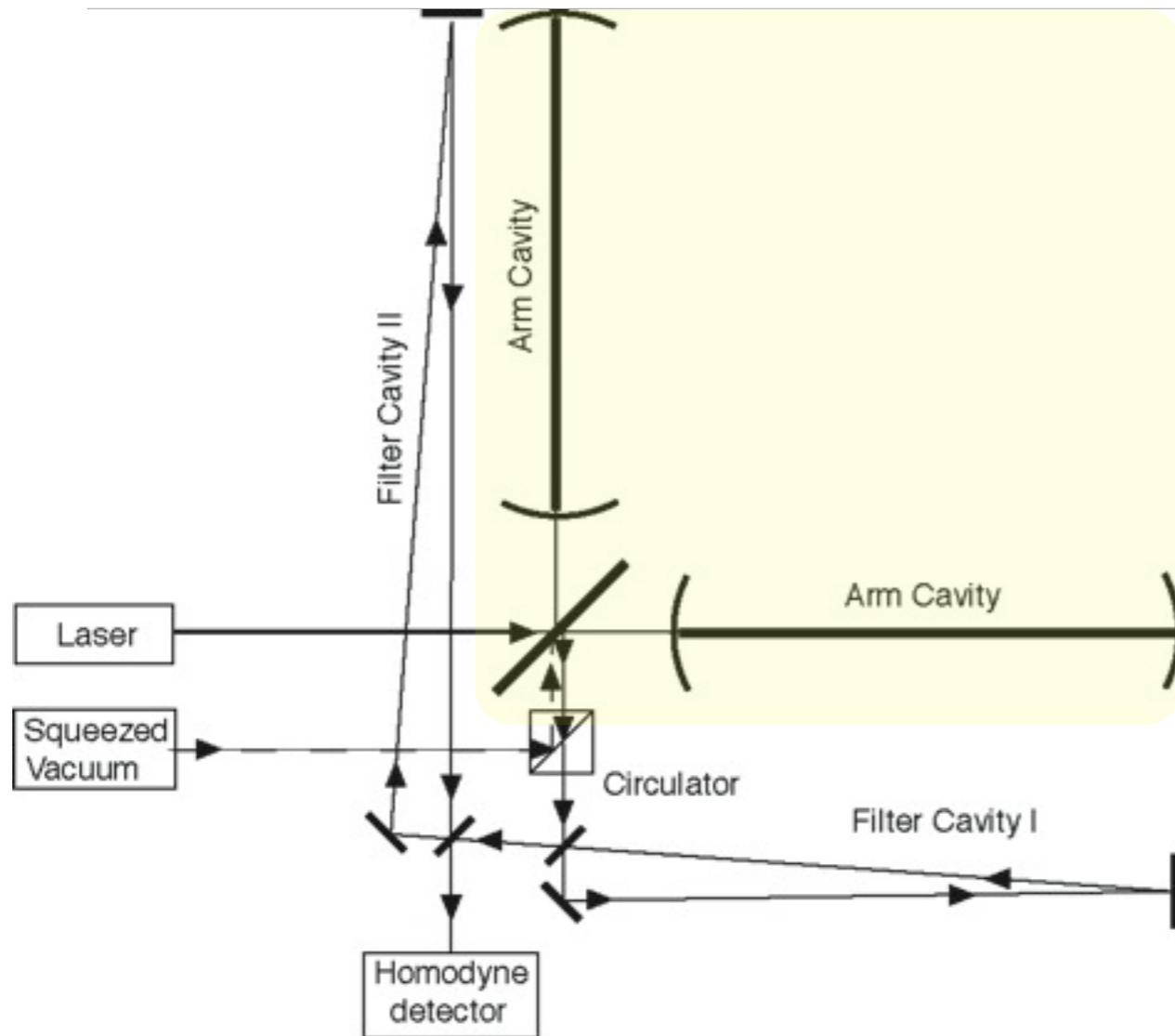
amplitude modulation
(photon kicks to mirrors)

phase modulation
(mirror motion plus shot noise)

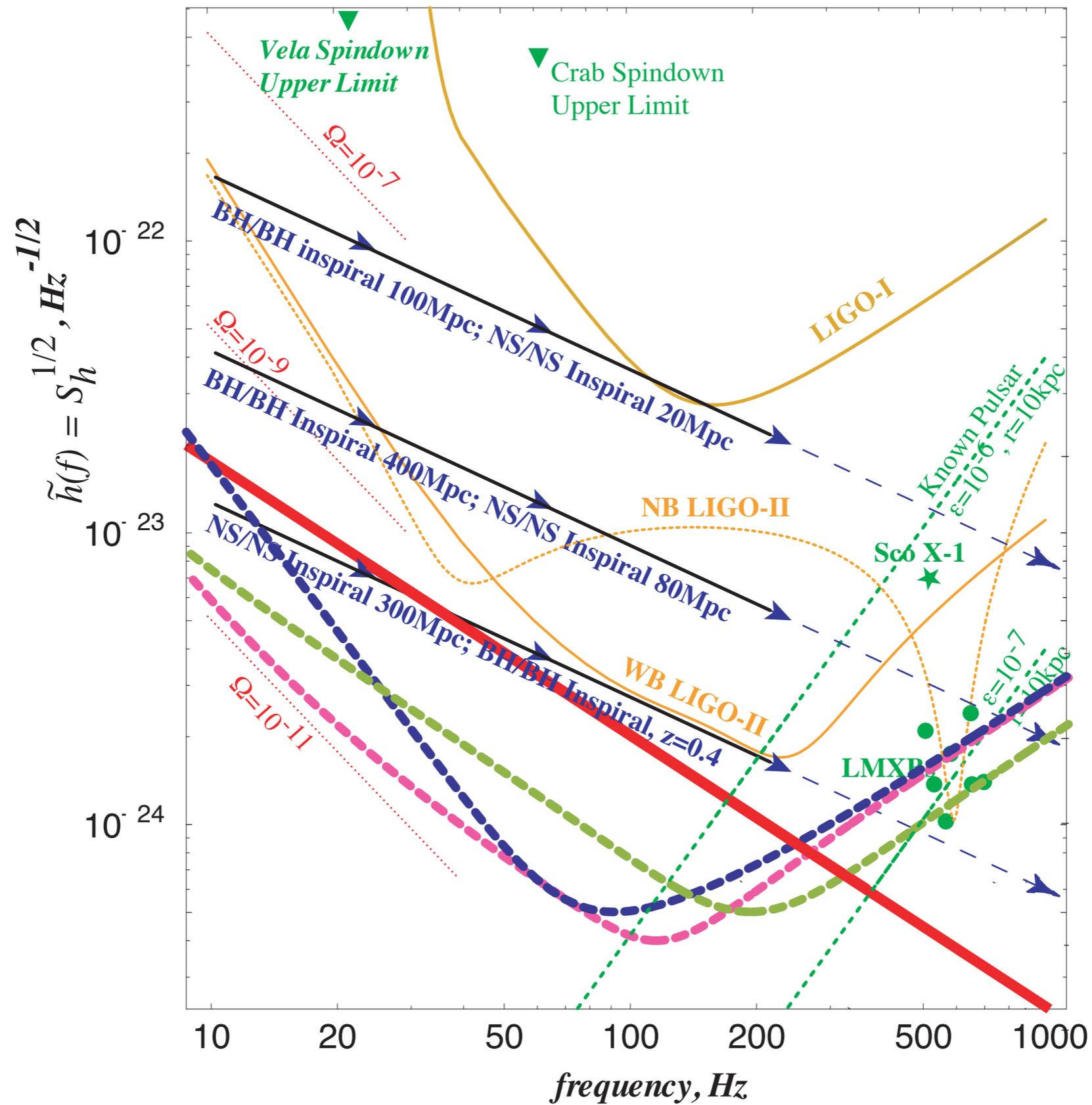
mirror motion: GW-induced & kick induced

Radiation-Pressure Noise
canceled when **combination**
between amplitude and phase is
measured

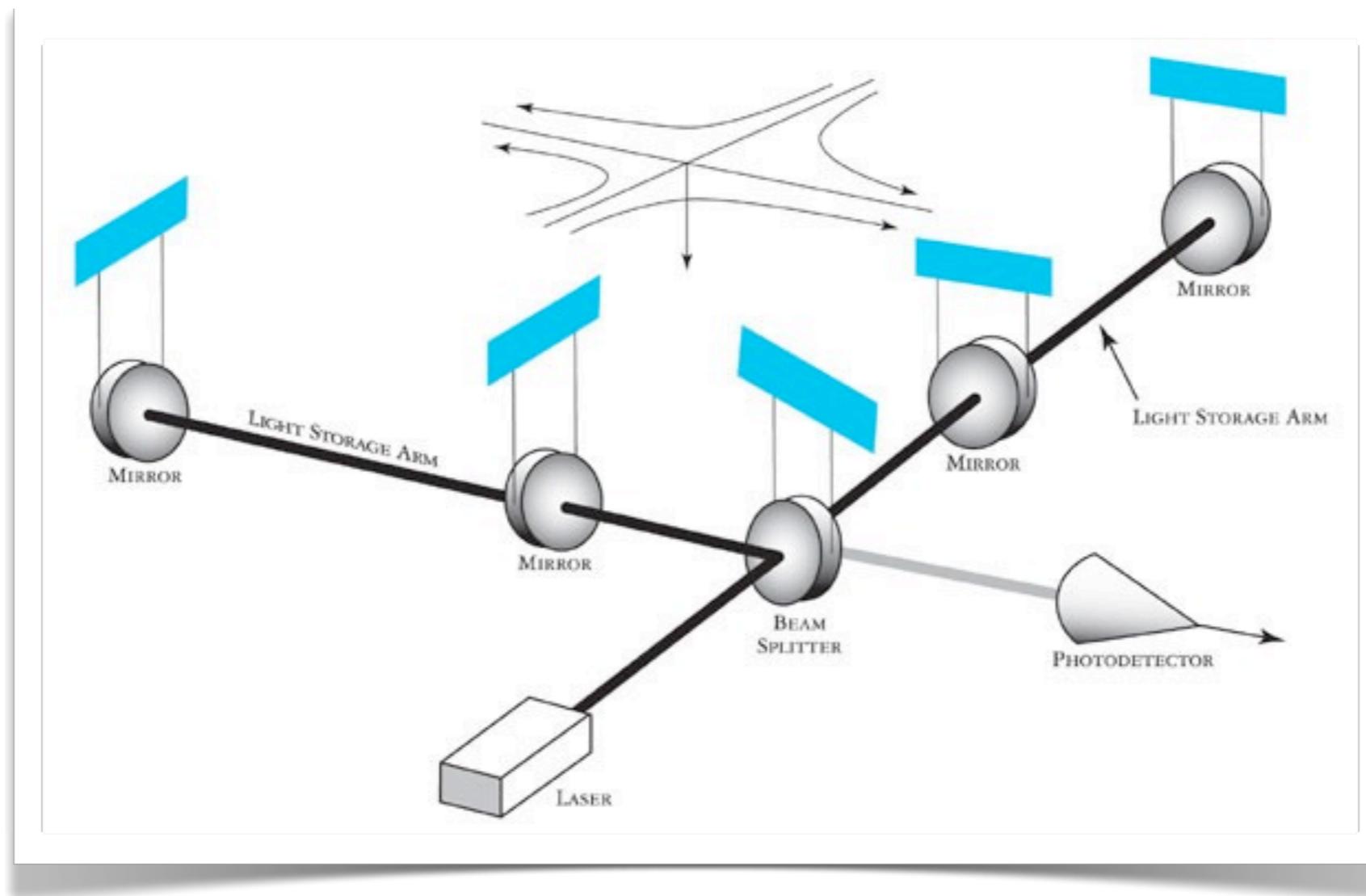
Designs become more complicated



Noise spectra of 1, 2 and 3G detectors



LIGO exploration of gravity decoherence



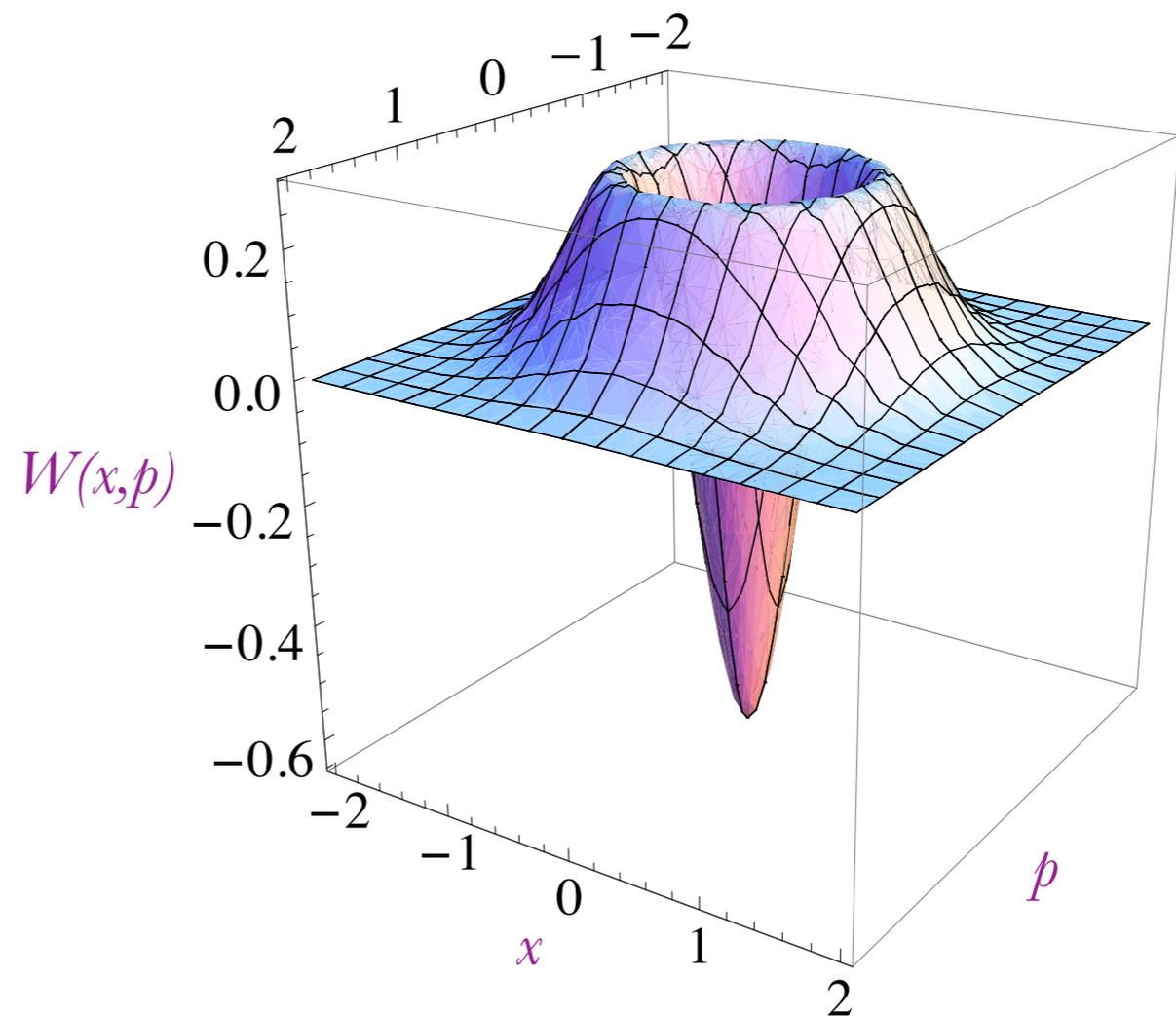
- Prepare quantum superposition state & observe how fast it becomes classical
 - survival time due to standard quantum mechanics & environmental decoherence: ~ 100 ms
 - gravity decoherence time: could be far less than 1 ms because mirrors are heavy

Preparation of non-classical quantum states

- We can also prepare exotic mirror quantum state without classical counterparts.
- **Wigner function**: *best analogy* to classical probability distribution of (x,p)
- Obtainable through measurements of $(a x + b p)$



CT image of a brain



Mirror state with non-positive *Wigner Function*

Summary

- Quantum Mechanics has been successful in the microscopic world, do they influence the macroscopic world?
- Yes! Although LIGO mirrors are heavy (10 kg), their quantum uncertainties will seriously affect sensitivity in the near future.
- Ways can be designed to circumvent those uncertainties.
- We can use LIGO to explore quantum mechanics of macroscopic objects